ABSTRACT

Nowadays, traditional methods of education, in particular in mathematics education, do not reply all required educational needs, and intelligent educational methods play a sensitive role in easy studying and understanding mathematics. But, lack of necessary facilities in some outlying schools, disinterestedness or debility of some teachers on learning new methods in education, are among some problems that make using these methods impossible. In this research, a new method for education, independent of the existing intelligence ones, especially in mathematics, is introduced. Then it has been exerted on two different groups of students at two different bases in one of the frontier provinces in Iran. Then the results are compared through inferential statistical methods with another two groups at same bases that have been educated with usually scholastic methods. Using statistical analysis shows that mathematics education through the introduced method has a competent role in amendment of mathematical problems of students.

KEY WORDS: Generalization method, Mathematics Education, Socratic Method, Team working method.

INTRODUCTION

Understanding the mathematical concepts is often difficult for students and most of them are usually disgust of learning mathematics. The absence of mathematics in their daily life changes the mathematics to a futile and harsh science for them. Most students have the ability to understand mathematical concepts, but in the argument stage which some connections between different concepts would be required, they are confronted with defects (Shalini, 2004). In this case, the reactions of students are different. Some of them are impotent in the first time, but if they have a second chance later, they will succeed without any great effort. A number of them can solve the problems for the first time, but when encounter the same problems for the next stage, they will be incapable of solving them, and at last there are some ones who can resolve the problems with several techniques (Golijani-Moghadam, et al., 2012). A useful method in mathematics education is a method that can be applicable for all three groups of them. In fact, according to the subjects planted in (Resnikoff and Wells, 2011) reforming the methods of education of mathematics is among the basic needs of every country that wishes to progress. In this work, the researcher having tried to introduce a new method using simultaneously three educational methods in teaching mathematics, and then compare the results with usual instruction method which today is established in most schools by using simple devices such as chalk and board (Sungur and Tekkaya, 2006).

Some kinds of Education Methods:

Socratic Method, Team working Method and at last Generalization Method are three general methods in education which all of those, based on the human intelligence and can cause the ability and opportunity for teachers and trainers to awake the dormant talents of students. In the Socratic Method, the thought navigation of the student is increased, while the aim of Generalization Method is increasing the ability of applying the results to different contexts and mathematical models (Bransford, et al., 2000) and at last the Team Working method uses some psychological characteristics of the students in teaching. In this method, the students should discuss with their more clever classmates whom is determined by their teacher, about the problems which they are not permitted to ask the teacher about them directly. A number of researchers have debated the important role of these skills of education in science (Dreistadt, 1968).
**Socratic Method:**

As the Longman Advanced American Dictionary explains, Socrates (469-399 B.C.) was a Greek philosopher from Athens, who was the teacher of Plato and is known for developing a method of examining ideas according to a system of questions and answers, the method that teacher does not give information directly, but instead asks the students a series of questions, as a way of directing the students to improve their thinking and knowledge. In his method, the subject was divided into small sections, and then the teacher planned some questions each of which corresponds to one of those small sections and its answer is just the same small part in the lesson, then asked them from one student to another respectively (Parsian, 2011). Socrates believed that among the factors that could cause the students to learning is the sharing their passions for knowledge. Students generally are incapable of understanding abstract concepts and there are several methods to help them. Each of these methods is an appropriate device for one age level and has its own characteristics. For example, analogical reasoning is suitable for primary school aged (Siegler and Svetina, 2002) but Socratic Method is one of the ways that is appropriate for all ages (Parsian, 2011). A noteworthy point is considering educational notes in planning the questions. Matching the scientific ability of students with the range of difficulty of questions is one of the most basic points which if fails to follow then eliminates the student’s confidence. The logical questions which asks by the teacher, resolution of questions, giving students adequate confidence. The help of the teacher should not be so that the utilities of thinking and reasoning of students are failed to act. The teacher should not be arrested him any more than what is required. His method should be such that the student feels that the main job is done by him. Designing the auxiliary problems is a major function of the human brain. Designing the problem which is useful for solving another one, is a thin achievements of human wisdom and his conscience (Polya, 1957).

**Problem 1:**

The teacher asks student to solve the equation $\sqrt{x + \sqrt{x - 1}} + \sqrt{x - \sqrt{x - 1}} = 2$.

**Solve:** This problem is difficult for the students who know the decomposition of quadratics only. The aim of the teacher is to guide the students to the right path, because his (her) ability of decomposition of a quadratic is known for the teacher. Hence by choosing the auxiliary unknown $y = \sqrt{x + \sqrt{x - 1}}$ and then $y^{-1} = \sqrt{x - \sqrt{x - 1}}$ a quadratic equation is obtained.

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**Problem 2:**

The teacher asks student: How can a fisherman bring up from the river exactly six quarts of water when he has only two containers, a four quart pail and a nine quart pail, to measure with?

**Discussion and Solve:**

The student not knows yet how to measure exactly six quarts. But could be measure something else? If he can not solve the proposed problem, try to solve first some related problem. Could he derive something useful from the data? He could fill the larger container to full capacity and empty so much as he can into the smaller container, then he could get five quarts. He is working now as most people do when confronted with this puzzle. He starts with the two containers, he tries this and that, he empties and fills, and when does not succeed, he starts again, tries something else. He is working foreword, from the given initial situation to the desired final situation, from the data to the unknown. He may succeed after many trails, accidently. From what foregoing situation could he obtain the desired final situation? He must have one quart in the smaller container! That’s the idea! But how can he reach the situation that he has just found? It is easy to recognize that we have one quart in the larger container. He fills the large container to full capacity, and pour from it four quarts into the smaller container and then into the river, twice in succession. He came eventually upon something already known. Other examples are introduced in (Parsian, 2014).

**Team Working Method:**

Today information technology has a great role in the development of educational objectives in developed countries. Rapid exchanges that take place, provides the availability of information around the country and make available for students an equal education. Internet has increasingly been used to provide significant resources for all of the students in various branches of science. According to (Shalini, 2004) the students of mathematics also use internet to learn mathematics and to gain knowledge about the development of this branch of science, as well as their teachers which internet is a significant resource of concepts for them. In USA, a large scale experiment introducing and evaluating intelligent tutoring in an urban high school setting. They built an intelligent tutor, called PAT that supports this curriculum and has been made a regular part of ninth grade
algebra in three Pittsburgh schools. The results of this intelligent system have shown that comments of teachers that working in the computer lab with PAT, activate the students whom are confronted with difficulties in the normal classrooms (Goljani-Moghadam, et al., 2012). Also the researchers which are performed a research in smart schools field in Malaysia believed that the smart school education program is a systematically designed program that integrates teaching and learning with IT applications, include computer based teaching and web-based learning. The aim of these smart schools is based on teaching-learning principles, and practice to develop and produce future generations who are technologically literate, who can think critically and creatively and are able to manage and apply knowledge effectively as well as innovatively (Ya’acob et al., 2005). But unfortunately such cases are not true for poor or developing countries. The limited facilities at most schools and low interest of the teachers of mathematics in using the internet for mathematical purposes are the problems that disrupts the effectiveness of this approach (Parsian, 2011). Therefore, a method that can produce the same goals, although in a limited range is necessary. In fact if the deficiencies of the educational programs, teachers and books are ignored and the precision and accuracy to carry out them are supposed without any defect, yet there is not a good opportunity for students which are not usually at a same scientific level, to thinking in the classrooms. So one of the methods that can be used in these cases, is using of the opportunities that students are not in the classroom. The teacher, according to the number of his bright students, divided all of the students into groups of three, four or five persons, and appoints each of his bright students as a header of one of those groups. Then the necessary materials which are needed to train each of those groups describes by the teacher to the header. The header must work with the students of his (her) group according to the teacher’s training manual. Decreasing the differences in academic level of students is the minimum result of this method, and if done with the precision and subtlety, this difference can lead to an acceptable level (Parsian, 2011).

**Generalization Method:**

In this method, the teacher or one of the students raises a theorem, and then students try to justify it with the teacher guidance or propose a generalized form of the theorem with the same assumptions and then offer its proof. Perhaps most of the great mathematicians have benefited of this way in their works. Major parts of mathematics are consecutive generalizations, but the clues to these generalizations are not clear, because of their owners’ skills. In fact, mathematics is thinking (Mason, et al., 1982) and so students should also learn the ways of thinking, not just a set of concepts, definitions and theorems. A motive force is needed to stimulate the student to thinking, which could be a matter of geometry, number theory, or any other branch of mathematics. To some believes of mathematicians, problems are the heart of mathematics and to some others, those are like the veins that carry blood to the body of mathematics. Moreover, from a point of view, mathematics is indeed solving the problems which are one of the human mental phenomena (Polya, 1981). On the other hand, as in (Polya, 1957) teaching mathematics is an art such as active sports like skiing, swimming and playing piano. This art can be taught only by imitation of a good model. This art is no magic key to unlock all doors but there are guidelines that will familiarize us with some useful doors. In the depths of human soul, a greater tension lies which is seeking to find some magical methods to open each site. This tendency in most of ours is secret and some legends and writings are created about them by some philosophers in their literary works. Descartes wanted to seek a useful method for all problems (Bell, 1986). Leibniz also planned to find a general method where can explained all of the intellectual truths by calculations and symbols (Bell, 1986). But these attempts were abortive such as the change some low-value metals into gold. There are some hopes which are still remaining. In fact, unattainable ideals were not so useless, for example who has achieved so far to the sun? But who can claim that the sun is useless?

Here are some examples that happened to the researcher in the classroom and were discussed by the students:

**Problem 3:**

A farmer will propose to build four water storage cubic containers such that the lengths of their edges are integers and the volume of large one is the sum of the volumes of the smaller ones. What are the lengths of their edges? **Solve and discussion:** The student who was familiar with quadruple Platonic, showed that \(3^3 + 4^3 + 5^3 = 6^3\) and deduced that the lengths of their edges can be 3,4,5 and 6. Another student generalized the recent result by multiplying the sides of identity by \(l^3\) for \(l \in N\) and deduced that the lengths of their edges can be \(3l, 4l, 5l\) and \(6l\) for all \(l \in N\). The third student. considered the equation \((3l - x)^3 + (4l + x)^3 + (5l - x)^3 = (6l - x)^3\) for unknown \(x\) and deduced \(x = 3l\), so another solution of the problem can be \(l, 6l, 8l\) and \(9l\) for all \(l \in N\). Repeating the method of the third student gives us some infinitely many independent solutions.

**Problem 4:**

The second power of one, five and six, are limited to the same numbers respectively! Are there any numbers with two digits which the second powers of those tend to the same digits? How about the numbers with three digits? **Solve:** A student who knew the rule of computing the second power of the numbers with two digits
which tend to five, offered the number 25 as a solution with two digits. For three digits the third student assumed that $a = \sqrt[2]{25}$ be such that $a^2 = \sqrt[2]{25}$ and by a simple calculation showed $x = 6$ and $a = 625$. In fact, those students show that the Diophantine equation $c_n^2 - c_n = a_n \times 10^n$ has a solution for $n = 1, 2$.

Now the following problem immediately comes to mind,

**Problem 5:**

Is the following Diophantine equation $c_n^2 - c_n = a_n \times 10^n$ has a solution for all $n \in N$?

**Discussion and solve:**

At this stage the sudden appearance of this equation for current students is no wonder. Because they have a prospect in their minds. But the solution in this case does not so simply. For solving this problem we use a note in the above students' dialogue! Note that the second solution came from the first and the third from the second. So we consider the equation $c_n^2 - c_n = a_n \times 10^n$ for which $c_n$ is an $n + 1$ digits sequence. The assumption $c_{n+1} = \sqrt[n]{a_n}$ and two previous equations with a simple calculation show that $10^n \times x^2 + (2a_n - 1)x - 10a_n + a_n = 0$. If $c_n$ tends to, then a necessary and sufficient condition for the existence of a solution for the last equation, is the existence of solution for $(2a_n - 1)x + a_n = 10u$. But this is a simple result of the theory of linear Diophantine equations, since $\text{gcd}(2a_n - 1, 10) = 1$ (Niven and Zuckerman, 1972). Now since $a_n, c_n$ have been identified by mathematical induction, $x$ is obtained by solving the last equation.

**Statistical analysis:**

In 2014 we did a statistical analysis to study the effects of Socratic-Team working-Generalization triple method (STG) in progress of mathematics education. In the present research two classes $I_2$ and $I_3$ of second year high school and two classes $I_3$ and $I_6$ of third year high school all in one of the frontier provinces in Iran were selected as statistical societies. Traditional teaching in classes $I_2$ and $I_3$ and STG training in the classes $I_6$ and $I_3$ was conducted by a unique tutor. For this study, four mathematical tests were used. In the two first mathematical tests, the prior knowledge of students was evaluated. The questions of third and fourth tests which has been designed by specialists to assess the mathematical skills of students after applying the methods of traditional and STG education. The Table 1 is regulated after collecting data using descriptive statistics.

In Table 1, it is shown that marks’ means of traditional and STG instruction groups of second year high school are 14.1 and 14.6 respectively. The marks’ means of the students of third year high school are also 14.1 and 14.5. Therefore the marks’ means of STG instruction groups are greater than marks’ means of traditional instruction groups. In addition, a similar result is established for the variance of the groups. We will estimate a confidence interval for mean difference of 95 percent of the first class of students and such an interval for the students of grade two. Here, $N_1, N_2 > 30$ and the t-student distribution is consistent with the normal distribution approximately. The variance of the whole society is unknown. If $S_p, L$ and $U$ be defined as in (Larson, 1982), then the details of calculating of 0.95 confidence intervals of two provinces have been recorded in Table 2.

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<th>Table 1: Descriptive statistics of traditional and STG instruction groups in math</th>
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<th>Table 2: The comparison of means of Traditional and STG instructions groups</th>
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**Discussion and conclusion:**

STG instruction method can be used to elevate the learning and teaching processes. The results of this research have demonstrated that using this instruction methods play a basic role in deep and better learning in mathematics at high schools. In fact, STG method is a combination of three methods each of those extends a dimension of deep teaching and learning in mathematics. Team working method is a good cooperative for teacher in teaching mathematics and has the ability to increase the level of deep teaching and learning of students with the aid of their classmates, while the Socratic and Generalization methods use the mind of student
in content creation. In this method, as good as the students' awareness raise, the differences in their mathematical knowledge also decrease.

REFERENCES

