Solution For Traveling Salesman Problem Using Fuzzy Multi-Objective Linear Programming

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ABSTRACT

Traveling Salesman Problem (TSP) is an important problem in Artificial Intelligence and Operations Research domain. The problem has been investigated under different headings and solved with various approaches including soft computing and linear programming. The conventional linear programming deals with crisp parameters. However, information available in real life system is of vague, imprecise and uncertain nature. The impreciseness and uncertainty aspects are handled using Fuzzy Sets to obtain optimal solutions. Multi-Objective Linear Programming effectively deals with flexible aspiration levels or goals. Fuzzy Multi-Objective Linear Programming enhances the effectiveness of solutions with acceptable solutions through fuzzy constraints. In this work, Fuzzy Multi-Objective Linear Programming is used for solving TSP with vague and imprecise parameters. An example of TSP with multiple objectives and imprecise parameters is also discussed.

Key words: Traveling salesman problem, fuzzy multi-objective linear programming, multiple objectives, vague parameters, aspiration levels.

INTRODUCTION

Traveling Salesman Problem (TSP) is well-known NP-hard combinatorial optimization problem. It represents the class of problems in which least-cost sequence is found for visiting set of cities, starting and ending at the same city such that each city is visited exactly once. The desire of decision maker where least time span or distance is significant formulates TSP as Multi-Objective Problem. Considering TSP as Multi-Objective Optimization Problem, each objective function is represented in distinct dimension. To decide Multi-Objective TSP in optimality means determining k-dimensional points pertaining to space of feasible solutions of problem and minimum possible values according to all dimensions. The permissible deviation from specified value of structural dimension is also considerable because salesman can face a situation in which objectives are achieved completely. There are a set of alternatives from which he selects one that best meets his aspiration levels. Conventional programming approach does not deal with this situation.

2. Literature Survey:

Branch and Bound approach was used to solve TSP with two sum criteria [6]. For max ordering TSP, 2-opt and 3-opt heuristics was used [7]. Sigal (1994) proposed Decomposition approach for solving TSP with respect to two criteria of route length and bottlenecks where both objectives are obtained from same cost matrix. Branch and Bound method with multiple labeling schemes was used to keep track of possible Pareto optimal tours [14]. An E-constrained based algorithm for Bi-Objective TSP was suggested by Melamed and Sigal (1997). An Approximation algorithm with worst case performance bound was proposed by Ehrgott (2000). Hansen [8] applied Tabu Search algorithm to Multi-Objective TSP.
Borges and Hansen [5] used weighted sums to study global convexity for Multi-Objective TSP. Jaskiewicz [9] proposed Genetic Local Search which combines ideas from Evolutionary Algorithms, Local Search with modifications of aggregation of Objective functions. Two Phase Local Search procedures to tackle Bi-Objective TSP was proposed by Paquete and Stützle [12]. During first phase, a good solution to one single objective is found by using an effective single objective algorithm. This solution provides starting point for second phase in which Local Search algorithm is applied to sequence of different aggregations of objectives, where each aggregation converts Bi-Objective problem into single objective one. Yan et al. [15] used an Evolutionary Algorithm to solve Multi-Objective TSP. Angel et al. [2] proposed a Dynamic Search algorithm which uses Local Search with an exponential sized neighborhood that can be searched in polynomial time using Dynamic Programming and Rounding technique. Paquete et al. [12] suggested Pareto Local Search method which extended Local Search algorithm for single objective TSP to Bi-Objective case. This method uses an archive to hold non dominated solutions found in the search process. Sepideh Fereidouni [13] used a fuzzy multi-objective linear programming. Arindam Chaudhuri, Kajal De [3] used a Fuzzy multi-objective linear programming for traveling salesman problem.

Furthermore, in TSP salesman takes decision of selecting an optimal and feasible route between any couple of cities on basis of expected measures. In most real world problems it is not possible to have all constraints and resources in exact form rather they are in expected or vague form.

This leads to use of Fuzzy Logic which enables us to emulate human reasoning process and make decisions based on vague or imprecise data. Fuzzy Programming gives methodology of solving problems in Fuzzy environment. An ideal solution method would solve every TSP problem to optimality, but this is not practical in most large problems. While advances have been made in solving TSP, these advances have been obtained at expense of intractable and complex nature of solutions. It is required to meet aspiration level of decision maker under which current optimal solution remains still optimal and feasible.

In this work a paradigm is developed which deals with vague parameters and achieve certain aspiration level of optimality for Multi-Objective Symmetric TSP by transforming it into a Linear Program using Fuzzy Multi-Objective Linear Programming (FMOLP) technique. The Route selection of problem is done by exploiting aspiration level parameters. The decision maker introduces tolerances to accommodate vagueness.

By adjusting tolerances, range of solutions with different aspiration level are found from which decision maker chooses one that best meets his satisfactory level within given domain. This paper discusses, the concept of Fuzzy programming, as well as illustrates FMOLP for TSP. A simulation example is given and conclusions are presented.

3. Fuzzy programming:

Fuzzy Multi-Objective Linear programming is illustrated to derive algorithm of TSP. It is based on the concept of Fuzzy Logic and Multi-Objective Linear Programming which is discussed in the next two subsections.

3.1 Fuzzy membership functions:

Fuzzy Logic introduced by Zadeh [16] is an extension of conventional two state logic which has been used to handle partial truth; that is, truth values between completely true and completely false. This logic underlies modes of reasoning which are approximate in nature. Linguistic term better represents subjective viewpoint of decision makers in more intuitive way and natural language format. The significance of Fuzzy Logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature.

Fuzzy Sets use linguistic variables rather than quantitative variables to represent imprecise concepts. A membership function of Fuzzy Set viz., Fuzzy Membership function is mapped on interval [0,1] which is an arbitrary grade of truth. The notation for Fuzzy Membership function $\mu_A(X)$ set $A$ is $\mu_A : X \rightarrow [0,1]$.

4. Multi-objective linear programming:

The first formal representation of Linear Programming Problem and an efficient technique for solving it was developed by Dantzig. The general Linear Programming model for maximization problem proposed by Dantzig is given by,

\[
\max \ Z = \sum_{i=1}^{m} c_i x_i
\]

Subject to $\sum_{i=1}^{n} a_{ij} x_i \leq b_j, j = 1,2,\ldots, m$

$x \geq 0$

where, $Z$ is objective function, $x_i$ are decision variables, $m$ is number of constraints, $n$ is number of decision variables and $b_i$ are given resources. Linear Programming model is solved by different methods such as Graphical method, Simplex method etc. Linear Programming is limited by the fact that it can deal only with single objective function and does not incorporate soft constraints. Multi-Objective Linear Programming is an extension of Linear Programming. It was introduced by Zeleny [17]. A general Linear Multiple Criteria Decision Making model can be represented as follows:
Find a vector $X$ such that $X^T = [x_1, ..., x_n]$ which maximize $k$ objective functions with $n$ variables and $m$ constraints as:

$$\text{max } Z_i = \sum_{j=1}^{n} c_{ij} x_j; i = 1, 2, ..., k$$  \hspace{1cm} (2)$$

subject to $\sum_{i=1}^{n} a_{ij} x_i \leq b_j; j = 1, 2, ..., m$

where, $c_{ij}, a_{ij}$ and $b_i$ are given crisp values. In precise form, multiple objective problems can be represented by following Multi-Objective Linear Programming model:

Optimize $Z= CX$

subject to $AX \leq b$

where, $Z = [z_1, ..., z_n]$ is vector of objectives, $C$ is $K \times N$ matrix of constants and $X$ is $N \times 1$ vector of decision variables, $A$ is $M \times N$ matrix of constants and $b$ is $M \times 1$ vector of constants.

5. Fuzzy multi-objective linear programming:

The concept of decision making in Fuzzy environment involving several objectives was first proposed by Bellman and Zadeh [4]. Zimmermann [18] applied their approach to vector maximum problem by transforming FMOLP Problem to single objective linear program. Considering the following Multi-Objective Linear Programming model,

$$\text{max } Z= CX$$

subject to $AX \leq b$

Adopted Fuzzy model by Zimmermann is given by,

$$\text{Max } Z^0 \leq CX$$

Subject to $AX \leq b$

Where, $Z^0 = [z^0_1, ..., z^0_n]$ are goals or aspiration levels; $\geq$ and $\leq$ are fuzzy inequalities that are fuzzifications of $\geq$ and $\leq$, respectively. For measurement of satisfaction levels of objectives and constraints Zimmermann suggested simplest type of Membership function given by,

$$\mu_{ik} (C_k X) = \begin{cases} 0 & \text{if } C_k X \leq z^0_k - t_k \\ 1 - (z^0_k - C_k X) / t_k & \text{if } z^0_k - t_k \leq C_k X \leq z^0_k, k = 1, 2, ..., n \\ 1 & \text{if } C_k X \geq z^0_k \end{cases}$$  \hspace{1cm} (6)$$

Another class of Fuzzy Membership functions suggested by Zimmermann has $\mu_{2i}(a_i X)$ for $i^{th}$ constraint as follows:

$$\mu_{2i} (a_i X) = \begin{cases} 0 & \text{if } a_i X \geq b_i + d_i \\ 1 - (a_i X - b_i) / d_i & \text{if } b_i \leq a_i X \leq b_i + d_i, i = 1, 2, ..., m \\ 1 & \text{if } a_i X \leq b_i \end{cases}$$  \hspace{1cm} (8)$$

d_i is admissible violation for fuzzy resource $b_i$ for $i^{th}$ constraint. These Membership functions express satisfaction of decision maker so they must be maximized. As a result objective function becomes,

$$\text{max}( \mu_{11} (C_1 X), ..., \mu_{ik} (C_k X), \mu_{21} (a_1 X), ..., \mu_{2m} (a_m X))$$  \hspace{1cm} (9)$$

According to Fuzzy Sets, membership function of intersection of any two or more sets is minimum Membership function of these sets. By virtue of this objective function becomes:

$$\text{min}( \mu_{11} (C_1 X), ..., \mu_{ik} (C_k X), \mu_{21} (a_1 X), ..., \mu_{2m} (a_m X))$$  \hspace{1cm} (10)$$

From above representation we have,

$$\text{maximize } CX \leq Z^0$$
\[ \alpha \leq 1 - (z_0^k - C_k X) / t_k^i ; k = 1, 2, \ldots, n \quad \alpha \leq 1 - (a_i X - b_j) / d_{ij} ; k = 1, 2, \ldots, m \] (11)

where, \( \tilde{\alpha} \) is overall satisfaction level achieved with respect to solution.

6. Fuzzy Multi-Objective Linear Programming Approach for TSP:

The most frequently considered objective of TSP is to determine an optimal order for traveling among all cities so that total cost is minimized. Consider the situation when decision maker has to determine optimal solution of TSP with minimized cost, distance, time, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time. The individual objective functions can be the link from city \( i \) to \( j \)

\[ x_{yj} = \begin{cases} 1, & \text{city}(i) \rightarrow \text{city}(j) \\ 0, & \text{otherwise} \end{cases} \] (12)

Let \( c_{ij} \) be the cost of traveling from city \( i \) to \( j \) overall cost of particular route is sum of costs on links comprising the route. Since, decision maker has to minimize overall traveling cost the goal can be set for total estimated cost of entire route for TSP denoted by \( z_1^0 \). But there can be situations when estimated cost doesn’t meet and so decision maker can set tolerance for estimated cost. Denoting tolerance against this goal as \( t_1 \), objective functions for minimization of cost is given as follows:

\[ z_1 : \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{yj} \leq z_1^0 \] (13)

Let \( d_{ij} \) be the distance from city \( i \) to \( j \) and \( z_2^0 \) be the corresponding aspiration level for objective functions for minimization of distance & \( t_2 \) be tolerance, then objective functions takes following form:

\[ z_2 : \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{yj} \leq z_2^0 \] (14)

Let \( t_{ij} \) be the time spent in traveling from city \( i \) to \( j \) and \( z_3^0 \) be the corresponding aspiration level for objective functions for minimization of total time & \( t_3 \) be tolerance, then objective function is written as follows:

\[ z_3 : \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{yj} \leq z_3^0 \] (15)

Let \( f_{cij} \) be the fuel cost spent in traveling from city \( i \) to \( j \) and \( z_4^0 \) be the corresponding aspiration level for objective function for minimization of total fuel cost and \( t_4 \) be tolerance, then objective function is written as follows

\[ z_4 : \sum_{i=1}^{n} \sum_{j=1}^{n} f_{cij} x_{yj} \leq z_4^0 \] (16)

Let \( vmc_{ij} \) be the vehicle maintenance cost spent in traveling form city \( i \) to \( j \) and \( z_5^0 \) be the corresponding aspiration level for objective function for minimization of total vehicle maintenance cost and \( t_5 \) be tolerance, then objective function is written as follows

\[ z_5 : \sum_{i=1}^{n} \sum_{j=1}^{n} vmc_{ij} x_{yj} \leq z_5^0 \] (17)

Let \( ta_{ij} \) be the traveling allowance for salesman spent in traveling form city \( i \) to \( j \) and \( z_6^0 \) be the corresponding aspiration level for objective function for minimization of total traveling allowance for salesman and \( t_6 \) be tolerance, then objective function is written as follows

\[ z_6 : \sum_{i=1}^{n} \sum_{j=1}^{n} ta_{ij} x_{yj} \leq z_6^0 \] (18)

Let \( twt_{ij} \) be the traveling wastage time for salesman spent in traveling form city \( i \) to \( j \) and \( z_7^0 \) be the corresponding aspiration level for objective function for minimization of total traveling wastage time for salesman and \( t_7 \) be tolerance, then objective function is written as follows

\[ z_7 : \sum_{i=1}^{n} \sum_{j=1}^{n} twt_{ij} x_{yj} \leq z_7^0 \] (19)

One important aspect is dependency of objective function on each other. They are mostly dependent, but determining exact form of dependency is complex process. The proposed frame work in all cases, if there is some feasible solution. These multiple objective functions are represented in vector form comparing multiple objectives with specified goal and tolerances. The membership functions to check their level of acceptability. A restriction is imposed that every city should be visited from exactly one of its neighboring city and vice versa, that is \[ \sum_{i=1}^{n} x_{yi} = 1, \forall j \] (20)

\[ \sum_{j=1}^{n} x_{yi} = 1, \forall i \] (21)

Route cannot be selected more than once, that is, \( x_{yi} + x_{yi} \leq 1, \forall i, j \) and non-negative constraints \( x_{yi} \geq 0 \). These constraints collectively are expressed in vector form and fuzzy membership functions are defined for all objective functions.
Finally, linear model is formulated FMOLP model using TSP objective functions, constraints and their corresponding membership functions. The model is solved be mixed integer Linear programming.

Simulation example:
The proposed FMLOP approach for TSP is analyzed with symmetric TSP, where salesman starts from his home city 0, visit three cities exactly once and comes back to his home city 0 by adopting route with minimum cost, time, distance covered, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time.

A map of cities listed along with their \((\text{cost, distance, time, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time})\) matrix in table-1, where \((c,d,t,f,c,v,m,c,t, w,t)\) represents \((\text{cost, distance, time, fuel cost, vehicle maintenance cost, traveling cost, traveling wastage time})\) respectively for corresponding pair of cities.

| Table1: The matrix for cost, distance, time, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time each pair of cities. |
|---|---|---|---|---|---|---|
| City | 0 | 1 | 2 | 3 |
| \((c,d,t,f,c,v,m,c,t, w,t)\) | \((0,0,0,0,0,0)\) | \((20,5,4,11,3,2,6)\) | \((15,5,5,10,3,3,4)\) | \((11,3,2,6,5,4,2)\) |
| \((0,0,0,0,0,0)\) | \((0,0,0,0,0,0)\) | \((30,5,3,15,5,5,2)\) | \((10,3,3,20,4,3,6)\) |
| \((15,5,5,10,3,3,4)\) | \((0,0,0,0,0,0)\) | \((20,10,2,15,4,5,1)\) | \((0,0,0,0,0,0)\) |
| \((11,3,2,6,5,4,2)\) | \((10,3,3,20,4,5,6)\) | \((20,10,2,15,4,5,1)\) | \((0,0,0,0,0,0)\) |

Let links \(x_{ij}\) be decision variables of selection of \(nk(i, j)\) from city 0 to 1. The objective functions \(z_1, z_2, z_3, z_4, z_5, z_6, z_7\) are formulated for \((\text{cost, distance, time, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time})\) respectively. Their aspirations level are set as 65,16,11,15,10,8 and 5 by solving each objective function subject to given constraints is TSP and their tolerances are decided as 5, 2, 1, 4, 3, 2 and 1. The corresponding objective function are as follows:

\[
\min z_1 = 20x_{00} + 15x_{02} + 11x_{03} + 20x_{10} + 30x_{11} + 10x_{12} + 15x_{20} + 30x_{21} + 20x_{23} + 11x_{30} + 10x_{31} + 20x_{32} \leq 65 \quad (22)
\]

Tolerance= \(t_1 = 5\)

\[
\min z_2 = 5x_{00} + 5x_{02} + 3x_{03} + 5x_{10} + 5x_{12} + 3x_{13} + 5x_{20} + 5x_{21} + 10x_{23} + 3x_{30} + 3x_{31} + 10x_{32} \leq 16 \quad (23)
\]

Tolerance= \(t_2 = 2\)

\[
\min z_3 = 4x_{00} + 5x_{02} + 2x_{03} + 4x_{10} + 3x_{12} + 3x_{13} + 5x_{20} + 3x_{21} + 2x_{23} + 2x_{30} + 3x_{31} + 2x_{32} \leq 11 \quad (24)
\]

Tolerance= \(t_3 = 1\)

\[
\min z_4 = 11x_{00} + 10x_{12} + 6x_{03} + 11x_{10} + 15x_{12} + 20x_{13} + 10x_{20} + 15x_{21} + 15x_{23} + 6x_{30} + 20x_{31} + 15x_{32} \leq 15 \quad (25)
\]

Tolerance= \(t_4 = 4\)

\[
\min z_5 = 3x_{10} + 3x_{12} + 5x_{10} + 3x_{10} + 5x_{12} + 4x_{13} + 3x_{20} + 5x_{21} + 4x_{23} + 5x_{30} + 4x_{31} + 4x_{32} \leq 10 \quad (26)
\]

Tolerance= \(t_5 = 3\)

\[
\min z_6 = 2x_{00} + 3x_{02} + 4x_{03} + 2x_{10} + 5x_{12} + 5x_{13} + 3x_{20} + 5x_{21} + 5x_{23} + 4x_{30} + 5x_{31} + 5x_{32} \leq 8 \quad (27)
\]

Tolerance= \(t_6 = 2\)

\[
\min z_7 = 6x_{00} + 4x_{02} + 2x_{03} + 6x_{10} + 2x_{12} + 6x_{13} + 4x_{20} + 2x_{21} + 2x_{23} + 2x_{30} + 6x_{31} + x_{32} \leq 5 \quad (28)
\]

Tolerance= \(t_7 = 1\)

The Fuzzy Membership function for cost, distance, fuel cost, vehicle maintenance cost, traveling allowance and traveling wastage time are illustrated below which are based on above equations.

\[
\mu(z_1) = \begin{cases} 
0 & \text{if } z_1 \geq 70 \\
1 - (z_1 - 65)/5 & \text{if } 65 \leq z_1 \leq 70 \\
1 & \text{if } z_1 \leq 65 
\end{cases} \quad (29)
\]

\[
\mu(z_2) = \begin{cases} 
0 & \text{if } z_2 \geq 18 \\
1 - (z_2 - 16)/2 & \text{if } 16 \leq z_2 \leq 18 \\
1 & \text{if } z_2 \leq 16 
\end{cases} \quad (30)
\]

\[
\mu(z_3) = \begin{cases} 
0 & \text{if } z_3 \geq 12 \\
1 - (z_3 - 11)/1 & \text{if } 11 \leq z_3 \leq 12 \\
1 & \text{if } z_3 \leq 11 
\end{cases} \quad (31)
\]
The above Fuzzy Linear Program and its variants are solved using MATLAB.

Conclusion:
In this work, Symmetric TSP is analyzed as Fuzzy problem with vague and imprecise decision parameters. For Multi-Objective TSP in uncertain environment, route selection is done by exploiting these parameters. The tolerances are introduced by decision maker to accommodate this vagueness. By adjusting these tolerances, a range of solutions with different aspiration levels are obtained from which decision maker chooses one that best meets his satisfactory level within given tolerances. FMOLP technique achieves $k - \alpha$ dimensional points according to decision maker’s aspiration level in Multi dimensional solution space.

References