Maximum Flow Problem in Fuzzy Environment

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ABSTRACT
In this paper we consider the problem of determining the maximum flow of a network. The problem of finding the path with maximum flow in a weighted directed network is very significant in the area of networking. In maximum flow problem, the weight of the edges is considered as the capacity of flow. In this paper an algorithm is proposed to find a path with maximum flow from source node to all other nodes of a network with trapezoidal fuzzy number as edge weight.

KEYWORDS: Maximal flow, Network, Label setting method, Fuzzy Number.

INTRODUCTION
Most of the problems that occur in industries can be treated as network problems such as flow distribution network, communication network, traffic network, and planning networks. Flow distribution network problem is very important in the field of operations research. The objective of the flow distribution network is to get maximum flow satisfying the edge capacities. The first maximum flow algorithm, was proposed by Ford and Fulkerson [4] in 1956. Applications and discussions of network flow problems were found in Foulds [5], Even [3], Lawler [6], and Tarjan [7]. In flow distribution network problem, parameters like distance, time and cost are certain in crisp environment. But in real situations there always exists uncertainty about the parameters. In such cases parameters can be represented by trapezoidal fuzzy numbers. In 1983, Dubois and Prade [2] dealt with ranking of fuzzy numbers. Abbasbandy and Asady [1] proposed new methods for ranking of fuzzy numbers. In this paper an algorithm is proposed to find a path with maximum flow of a network with trapezoidal fuzzy number as edge weight.

II. Preliminaries:
A. Fuzzy Numbers:
A fuzzy subset $\tilde{A}$ of the real line $\mathbb{R}$ with membership function $\mu_{\tilde{A}}: \mathbb{R} \rightarrow [0,1]$ is called a fuzzy number if
i. $\tilde{A}$ is normal, (i.e.) there exists an element $x_0$ such that $\mu_{\tilde{A}}(x_0) = 1$
ii. $\tilde{A}$ is fuzzy convex, (i.e.) $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1)\wedge\mu_{\tilde{A}}(x_2), \quad x_1, x_2 \in \mathbb{R}, \text{ for all } \lambda \in [0,1]$
iii. $\mu_{\tilde{A}}$ is upper continuous and
iv. $\text{Supp}\tilde{A}$ is bounded, where $\text{Supp}\tilde{A} = \{x \in \mathbb{R}: \mu_{\tilde{A}}(x) > 0\}$

B. Trapezoidal Fuzzy Number
The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ where $a_1 \leq a_2 \leq a_3 \leq a_4$ defined on $R$, is called the trapezoidal fuzzy number if membership function is given by

$$
\mu_X(x) = \begin{cases} 
\frac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\
1 & a_2 \leq x \leq a_3 \\
\frac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
$$

III. Algorithm:

Algorithm to find the path with maximum flow from source node to all other nodes of a network with trapezoidal fuzzy number as edge weight is as follows.

Step 1:
Let $N$ be the set of all nodes and $s \in N$ be the source node. Let $TLN$ be the set of temporary labeled nodes and $PLN$ be the set of permanently labeled nodes. Let $PLN = \varnothing$ and $TLN = N - \{s\}$. Let $F_i^k$ represent the capacity of a path with maximum flow from source node to node $i$ at $k^{th}$ iteration. Let $F_i^0 = (0,0,0,0)$ Let $C_{ij}$ be the weight of the edge $(i,j)$ and $C_{ij} = (0,0,0,0)$, if $(i,j) \notin E$. Let $k = 0$.

Step 2:
For all $i \in TLN$, if $(s,i) \in E$, then $F_i^0 = C_{si}$

Step 3:
Ranking of trapezoidal fuzzy number $F_i^k = (a_1, b_1, c_1, d_1)$ and $F_j^k = (a_2, b_2, c_2, d_2)$ is done as follows

If $\Re(F_i^k) > \Re(F_j^k)$ then $F_i^k > F_j^k$

If $\Re(F_i^k) < \Re(F_j^k)$ then $F_i^k < F_j^k$

If $\Re(F_i^k) = \Re(F_j^k)$ then $F_i^k = F_j^k$

where $\Re(F_i^k) = \Re(a_1, b_1, c_1, d_1) = \frac{1}{4}(a_1 + b_1 + c_1 + d_1)$

Step 4:
Using step 3, select a node $y$ from $TLN$, such that $F_y^k$ is maximum.
In case of tie, arbitrary selection is made. Remove the node $y$ from $TLN$ and include node $y$ to $PLN$.
Let $N^{k+1} = N$
If $TLN = \varnothing$, then
\{For all $i \in PLN$, $F_i^{k+1} = F_i^k$
Let $k = k + 1$ and go to step 6\}
Otherwise go to step 5

Step 5:
For all $(y,j) \in E$ compute $F_j^{k+1} = \max[ F_j^k, \min(F_y^k, C_{yj}) ]$

Else, if $(y,j) \notin E$ then $F_j^{k+1} = F_j^k$

For all $i \in PLN$, let $F_i^{k+1} = F_i^k$
Let $k = k + 1$. Go to step 4

Step 6:
Determination of path from source node $s$ to node $i$
Select a node $x$ arbitrarily from $PLN$.
Let $PLN = PLN - \{x\}$
Let $j = 0$ and Path $= < x >$
Step 7: 
If $F^k_j = F^k_{j+1}$ then go to step 9. Otherwise go to step 8

Step 8: 
Path = $N^k_j \oplus$ Path 
Let $x = N^k_j$. Go to step 7

Step 9: 
Let $j = j + 1$ 
If $k - j = 0$, then 
(Path = $s \oplus$ Path 
If $TLN \neq \phi$, go to step 6 
Else Terminate) 
Else go to step 7.

IV. Numerical Illustration:
To determine the path from node 1 to all other nodes with maximum flow capacity of a simple directed network given in figure 4.1. In this network, trapezoidal fuzzy numbers are considered as weight of the edges and the weight of the edges is given in table 4.1.

Table 4.1: Edge weights of the Network in figure 4.1

<table>
<thead>
<tr>
<th>Edge $(i, j)$</th>
<th>Weight of the edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 2)</td>
<td>10, 12, 16, 20</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>5, 8, 10, 12</td>
</tr>
<tr>
<td>(2, 3)</td>
<td>1, 2, 4, 5</td>
</tr>
<tr>
<td>(2, 4)</td>
<td>12, 16, 18, 20</td>
</tr>
<tr>
<td>(2, 5)</td>
<td>2, 5, 6, 8</td>
</tr>
<tr>
<td>(3, 5)</td>
<td>4, 6, 7, 10</td>
</tr>
<tr>
<td>(4, 5)</td>
<td>2, 8, 10, 16</td>
</tr>
<tr>
<td>(4, 6)</td>
<td>2, 4, 5, 7</td>
</tr>
<tr>
<td>(5, 6)</td>
<td>6, 8, 12, 14</td>
</tr>
</tbody>
</table>

Applying the algorithm proposed in section 3, the path with maximum flow capacity from node 1 to all other nodes is given in table 4.2.

Table 4.2: Path and its maximum flow

<table>
<thead>
<tr>
<th>Node i</th>
<th>Path with maximum flow from node 1 to node i</th>
<th>Maximum flow of the path</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1-2</td>
<td>(10, 12, 16, 20)</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>(5, 8, 10, 12)</td>
</tr>
<tr>
<td>4</td>
<td>1-2-4</td>
<td>(10, 12, 16, 20)</td>
</tr>
<tr>
<td>5</td>
<td>1-2-4-5</td>
<td>(2, 8, 10, 16)</td>
</tr>
<tr>
<td>6</td>
<td>1-2-4-3-6</td>
<td>(2, 8, 10, 16)</td>
</tr>
</tbody>
</table>

Conclusion
In this paper a new method is proposed to find a path with maximum flow from source node to all other nodes with trapezoidal fuzzy numbers as edge weights. An example is solved to illustrate the method.

REFERENCES