Ideal thermodynamic analysis of Stirling engine using Schmidt analysis

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ABSTRACT

Stirling Engines are external combustion engines which can use any fuel as heat source. They are one of the high efficiency engines which find many applications because of its design simplicity, low noise operation, diverse fuel usage, good mechanical balance etc. Schmidt analysis is an ideal isothermal thermodynamic analysis which is used to predict the performance of Stirling engine in the design stage. In this paper the Schmidt analysis had been carried out to predict the performance of an alpha stirling engine which is being fabricated. The analysis is used to refine the actual fabrication and optimize the power output from the engine.

KEYWORDS: Schmidt analysis, Stirling cycle, Stirling engine

INTRODUCTION

Stirling Engine was invented and patented by a Scottish engineer named Sir Robert Stirling in the year 1816. Stirling engine is theoretically one of the most efficient thermodynamic engines for heat to work conversion. Stirling engines are external combustion engines that can use any form of heat source such as exhaust gas from I.C engines, boilers, solar thermal energy, biomass combustion to name a few. The operating temperature of the stirling engine is between 650°C to 800°C, which makes it ideal for waste heat recovery and solar thermal applications.[1].
Configuration Of Stirling Engines:

Stirling engine configuration can be of three types namely Alpha, Beta and Gamma [2]. Of these Alpha configuration is simple with two pistons housed in two separate cylinders of inline, parallel or V configuration. Beta configuration has single cylinder for displacer and power piston with relatively complex arrangement. Gamma configuration has separate piston and displacer in different cylinders with simple arrangement.[3] Alpha configuration is selected for this study because of its relatively simple design and suitability for the intended application.

A. Alpha Configuration:

Alpha configuration shown in figure 1 has two pistons and no displacer. The two pistons namely hot piston and cold piston are housed in two separate hot and cold cylinder. Hot and cold cylinders are connected by a passage through a regenerator. These two pistons move in same direction to provide constant volume heat absorption or heat rejection. The compression and expansion is taking place when one of the pistons stop moving and the other piston continues to move with an increase or decrease in volume or pressure. Compression work is done by cold piston and expansion work by hot piston[1]

\[ V_{dc} \text{ Dead volume of compression cylinder} \]
\[ V_t \text{ Total momental volume} \]
\[ V_r \text{ Regenerator volume} \]
\[ V \text{ Volume ratio} \]
\[ W_e \text{ Indicated work in expansion cylinder} \]
\[ W_c \text{ Indicated work in compression cylinder} \]
\[ W \text{ Total indicated work} \]
\[ Y_{de} \text{ Dead volume ratio of expansion cylinder} \]
\[ Y_{dc} \text{ Dead volume ratio of compression cylinder} \]
\[ Y_r \text{ Dead volume ratio of regenerator} \]
\[ \alpha \text{ Crank angle} \]
\[ \eta_{engine} \text{ Thermal efficiency of the engine} \]

Fig. 1: Alpha configuration Stirling engine

II. Stirling Cycle:

Stirling cycle is defined as a ideal reversible thermodynamic cycle with two isothermal process with compression and expansion and two constant volume process with heating and cooling[4].

Fig. 2: Stirling cycle process

The figure 2 shows stirling cycle with isothermal and constant volume process. The working fluid is
compressed isothermally in the process 1. In the constant volume process 2 heat is added. Expansion takes place isothermally in the process 3 and constant volume heat rejection takes place in the process 4. [4]

The ideal thermodynamic Stirling cycle has the following advantages over theoretical Carnot cycle.

1. The efficiency of an ideal stirling cycle with regeneration is equal to Carnot efficiency, which has maximum possible theoretical efficiency of all heat engines operating between same temperature limits.
2. The two constant volume process of stirling engines increase the work output with reasonable increase in pressure and volume unlike Carnot cycle where pressure and volume increase are very high [2]
3. Of all heat engines operating between same pressure, temperature and volume ratios stirling engine has high mechanical efficiency [5]

In actual stirling engines the mass of gas in the dead volume is significant which undergoes isothermal compression and expansion process. This resulted in minimum loss of work per cycle compared with theoretical results [4]

**III. Schmidt Analysis:**

Schmidt analysis is thermodynamic analysis of Stirling engine by considering sinusoidal volume variations of the engine. It is a first order analysis which is very simple and does not take into account any losses that takes place in actual process. It also follows the ideal assumptions of isothermal compression and expansion as well as perfect regeneration. So it remains close to ideal cycle but more realistic in approach by taking into account the sinusoidal volume variations. [6]

**Assumptions for Schmidt analysis of Alpha stirling engines [6]**
1. All the process in the cycle are assumed to be reversible
2. Perfect regeneration is assumed. (i.e.) No heat losses in the regenerator
3. Working fluid obeys perfect gas laws (i.e.)\( PV = mRT \)
4. Mass of working fluid remains constant with no leakage.
5. The volume of cylinder varies sinusoidal with movement of piston.
6. No temperature gradient in heat exchanger
7. Cylinder wall and piston temperature remains constant
8. Engine rpm remains constant
9. Steady state conditions are assumed for the analysis
10. Heater and expansion space temperature remains constant (Isothermal)
11. Cooler and compression space temperature remains constant (Isothermal)
12. Regenerator temperature remains constant (Isothermal)

**IV. Schmidt analysis of alpha Stirling engine:**

**A. Thermodynamic equations:**

The initial calculation in solving Schmidt analysis is calculation of mass of working fluid. Perfect gas equation is used to calculate the various volumes of stirling engine.

The volume of compression cylinder and expansion cylinder with respect to various crank angles are determined. At Top dead centre of the piston the crank angle is assumed to be zero \(\alpha = 0\).

The instantaneous expansion cylinder volume \(V_e\) can be expressed as a function of expansion cylinder swept volume \(V_{sc}\), expansion cylinder dead volume \(V_{dc}\) and crank angle \(\alpha\).

\[ V_e = \frac{V_{se}}{2}(1 - \cos \alpha) + V_{de} \]  

Similarly the instantaneous compression cylinder volume \(V_c\) can be expressed as a function of compression cylinder swept volume \(V_{sc}\), compression cylinder dead volume \(V_{dc}\) and crank angle \(\alpha\).

\[ V_c = \frac{V_{sc}}{2}(1 - \cos \alpha - d\alpha) + V_{dc} \]  

Total instantaneous volume considering regenerator is given by

\[ V_t = V_e + V_c + V_r \]  

With the assumptions of constant pressure ,constant temperature and ideal gas working fluid the following equations can be obtained by ideal gas equations

\[ m = \frac{PV_e}{RT_e} + \frac{PV_c}{RT_c} \]  

The regenerator temperature is calculated by the following assumptions

\[ T_r = \frac{T_e}{2} + T_c \]

Where \(T_r = \frac{T_c}{2} + T_c\) T is Temperature ratio.
The volumetric ratio \( V \) is given by the ratio of stroke volume of compression cylinder to stroke volume of expansion cylinder.

\[
V = \frac{V_{cc}}{V_{ce}} \quad (7)
\]

The dead volume ratio is given by

\[
Y_{de} = \frac{V_{de}}{V_{ce}} \quad (8)
\]

for compression cylinder

\[
Y_r = \frac{V_r}{V_{ce}} \quad (10)
\]

The total mass of working fluid in the cylinder is given by

\[
m = \frac{P}{RT_c} \left( TV_e + \frac{2TV_c}{1+T} + V_c \right) \quad (11)
\]

Substitute Equation 1 & 2 in equation 11

\[
m = \frac{PV_{de}}{2RT_c} \left( H - J \cos(\alpha - F) \right) \quad (12)
\]

Where

\[
H = T + 2TY_{de} + 4TY_e + V + 2Y_{dc} \quad (13)
\]

\[
J = \sqrt{T^2 + 2TV \cos da + V^2} \quad (14)
\]

\[
F = \tan^{-1} \frac{V \sin da}{\sqrt{V + \cos da}} \quad (15)
\]

The engine pressure is calculated as

\[
P_e = \frac{V_{ce}}{2mRT_c} \left( H - J \cos(\alpha - F) \right) \quad (16)
\]

The mean pressure can be obtained from

\[
P_{mean} = \frac{1}{2mRT_c} \left( H - J \cos(\alpha - F) \right) \quad (17)
\]

The indicated work done by the engine in the expansion space \( W_e \) is calculated from the \( \int PdV_e \)

\[
W_e = \int P_{meanV_e}(\pi x \sin a) \frac{1 + \sqrt{1 - z^2}}{1 + \sqrt{1 + z^2}} \quad (18)
\]

The indicated work done by the engine in the compression space \( W_c \) is calculated from the \( \int PdV_c \)

\[
W_c = -\int P_{meanV_e}(\pi x \sin a) \frac{1 + \sqrt{1 - z^2}}{1 + \sqrt{1 + z^2}} \quad (19)
\]

The total indicated work done by the engine is

\[
W = W_e + W_c \quad (20)
\]

If the engine speed is \( n \) rps the the Total Indicated power from the engine is given by

\[
IP_{total} = W \cdot n \quad (21)
\]

The thermal efficiency of the engine is calculated as

\[
\eta_{engine} = \frac{W}{W_t} = 1 - T \quad (22)
\]

The efficiency is equal to Carnot efficiency which depends only on hot and cold cylinder temperature.

*Result from Schmidt analysis:*

![Actual Pressure-volume diagram of Stirling engine](image-url)
Figure 4 gives the relation of mean cylinder pressure with cylinder total volume. The maximum pressure of 99.05 MPa in the cycle occurs at crank angle of 160 degrees at crank angle of 340 degrees the minimum pressure of 49.54 MPa occurs. The Mean cylinder pressure is increasing from crank angle 0 degrees upto crank angle of 340 degrees and started to decline and reach the starting pressure of 50.63 at crank angle 360 degree.

Table 1: Results of Schmidt analysis of alpha stirling engine

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Maximum indicated power</td>
<td>92.2 Watts</td>
</tr>
<tr>
<td>Total Minimum indicated power</td>
<td>37.0 Watts</td>
</tr>
<tr>
<td>Mass of working fluid</td>
<td>0.001 kg</td>
</tr>
<tr>
<td>Engine speed</td>
<td>50 rps</td>
</tr>
<tr>
<td>Pressure of working fluid</td>
<td>1 bar</td>
</tr>
<tr>
<td>Mean pressure in the cylinder</td>
<td>0.562 MPa</td>
</tr>
<tr>
<td>Volume of expansion cylinder (Hot cylinder)</td>
<td>29452.4 mm³</td>
</tr>
<tr>
<td>Dead volume of expansion cylinder</td>
<td>1374.46 mm³</td>
</tr>
<tr>
<td>Volume of compression cylinder (Cold cylinder)</td>
<td>29452.4 mm³</td>
</tr>
<tr>
<td>Dead volume of compression cylinder</td>
<td>1374.46 mm³</td>
</tr>
<tr>
<td>Regenerator volume (Including dead volume)</td>
<td>9424.8 mm³</td>
</tr>
<tr>
<td>Volume ratio</td>
<td>0.45</td>
</tr>
<tr>
<td>Indicated work in expansion cylinder</td>
<td>25.1 Joules</td>
</tr>
<tr>
<td>Indicated work in compression cylinder</td>
<td>-11.3 Joules</td>
</tr>
<tr>
<td>Total indicated work</td>
<td>13.82 Joules</td>
</tr>
<tr>
<td>Dead volume ratio of expansion cylinder</td>
<td>0</td>
</tr>
<tr>
<td>Dead volume ratio of compression cylinder</td>
<td>0</td>
</tr>
<tr>
<td>Dead volume ratio of regenerator</td>
<td>0.172</td>
</tr>
<tr>
<td>Crank angle</td>
<td>90°</td>
</tr>
<tr>
<td>Thermal efficiency of the engine</td>
<td>54.9</td>
</tr>
</tbody>
</table>

This actual PV diagram in figure 4 differs from the ideal PV diagram given in figure 1 in the sense that there are no distinct points which separate the different process and the enclosed area of the curve is smaller than in ideal diagram. This clearly indicates the losses and the reduced work output from the actual engine.

The sinusoidal volume variations of hot cylinder, cold cylinder and total volume with respect to crank angles are given in figure 3. The volume is maximum at crank angle 320 degrees and minimum at 140 degrees. As the cold and hot pistons are phased at 90 degrees the cold volume and hot volume reaches maximum and minimum volumes at different crank angles. The cold volume and hot volume relation is given in figure 4. The curve gives the instantaneous cold volume, hot volume at any point of time.

Fig. 3: Volume variations with crank angle
Conclusion:
The Schmidt analysis give maximum engine power output of 92.2 Watts at 400 rpm and mean pressure of 0.562 MPa. The power obtained is theoretical power output and the actual power output is 30% of theoretical output due to losses like dead volume loss, leakage loss, thermal loss and windage loss. So the developed alpha engine will give a maximum output of 27.66 Watts. This analysis gives a fair prediction of the stirling engine output parameters and helps to determine the engine dimensions.

REFERENCES