Enhancement of Gaussian Noise Affected Images Using Modified Threshold Function in Curvelet Domain

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ABSTRACT

Background: An image is affected by noise during capture and transmission. This is because of the sensors and channel characteristic. In this paper modified thresholding function in curvelet domain is proposed to remove the noise present in the image and also preserving the image details. Objective: The modified threshold function will overcome the limitations of the discontinuities in hard threshold and reduces the permanent bias in the soft threshold method. Results: The proposed technique compared with wavelet hard, wavelet soft, wavelet modified threshold, curvelet hard and curvelet soft threshold techniques. The experimental results show the modified thresholding in curvelet domain is superior compared to other techniques. Conclusion: Proposed method removed more noise and preserve the details of the image as compared to other techniques and estimated image is approximately nearer to the original image.

INTRODUCTION

While capturing and transmission, images are affected by noise. The images may be affected by additive white Gaussian noise. This noise will corrupt the features of the image and during feature extraction process it misleads to recognize features in the image. Therefore image denoising is very much essential for any image processing applications. Earlier methods used for noise removal are spatial filters which are statistical filters (Gonzalez RC, Woods RE 2008). These filters will remove the noise to a certain extent, but at the same time smoothing the edges. After development of wavelet transform, they are used for noise removal. In wavelet domain to remove noise in an image, wavelet transform is taken for noisy image and coefficients of noisy images are divided into important and non important coefficients and each of these are modified based on rules. The basic functions for removing noise are hard and soft thresholding explained by Coifman and Donoho, and Donho and Johnstone (1995). The hard and soft threshold makes non important coefficients to zero. The hard and soft threshold functions have some limitations and they are discussed in the related work. The threshold function in curvelet domain is proposed in this work to overcome the limits of hard, soft threshold functions and wavelet transform.

2. Related Work:

Coifman and Donoho used hard threshold function in wavelet domain to remove noise (1995). But the hard threshold function is discontinuous in the whole wavelet domain and there are interrupted points at threshold. So the image constructed from this method contains residue of noise.

The soft threshold function used in wavelet domain to filter out noise coefficients by Donoho and Johnstone (1995). In soft threshold important coefficients having constant deviation result of this blurs the edges.

The modified threshold function is used in wavelet domain by Nasri and Nezamabadi-pour (2009), to overcome the limitations of hard and soft threshold function. Even though it overcomes the limitation of hard
and soft threshold, wavelet require more coefficients to represent edges. This limitation is overcome by the curvelet transform which represent edges with less number of coefficients. (Starck et al., 2000).

The hard threshold function used by Starck et al., (2000) for denoising in curvelet domain. Even though the curvelet will overcome the limitations of wavelet, because of hard threshold, it introduces residue of noise and the soft threshold will introduces constant deviation and smoothing the image.

The modified threshold function in the curvelet domain proposed in this work will overcome the limitations of the hard, soft threshold function and also the wavelet transform.

3. Curvelet Transform:

The fast discrete curvelet transform via wrapping technique (Emmanuel et al., 2005, 2006) is used in this work. It is simpler, faster and less redundant compared to first generation version. The curvelet transform is a multiscale transform like wavelet transform, its frame elements are indexed by scale and location parameters. Curvelet transform is based on anisotropic scaling, and wavelet is based on isotropic scaling. The elements of curvelet will follow the relation, \( \text{width} \approx \text{length}^2 \) (Starck et al., 2000).

The digital curvelet transformation is linear and takes as input Cartesian arrays of the form \( f[l_1,t_1],0 \leq l_1, t_1 < n \), and output as a collection of coefficients \( c^s(j,k,l) \) obtained by the digital analog to continuous curvelet (Donoho & Duncan, 1999).

\[
c^s(j,k,l) = \sum_{l_1,t_1,n} f[l_1,t_1] \phi_{j,l}[l_1,t_1] \quad (1)
\]

\( \phi_{j,l}[l_1,t_1] \) - Digital curvelet waveform.

The denoising process in case of curvelet is done by taking curvelet transform of the noisy image and apply a threshold to shrink the noisy coefficients, then apply inverse curvelet transform to obtain the denoised image.

4. Gaussian Noise:

The Gaussian noise will change each pixel of the image by a small amount from its original value (Ayushi et al., 2014). Gaussian noise is used in image denoising technique assume as zero-mean additive white Gaussian noise (AWGN) (Liu, and Lin, 2013). The noisy image \( I(x,y) \) can be expressed as

\[
I(x,y) = Io(x,y) + N(x,y)
\]

Where \( Io(x,y) \) indicates the original image, and \( N(x,y) \) indicates the signal-independent noise. 

The noise amplitude is given by a Gaussian distribution as shown in Eqn.3:

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}
\]

Where \( \sigma \) - noise standard deviation, \( \mu \) - mean value of distribution. For zero mean additive white Gaussian noise (\( \mu=0 \)), we have to calculate only \( \sigma \).

5. Threshold Functions:

5.1 Hard threshold:

In case of hard threshold, each absolute value of the coefficient is compared with a threshold, if it is less than the threshold, then the corresponding coefficient shrinks to zero, otherwise it retains the coefficient. It is given by Eqn.4

\[
f(x) = \begin{cases} 
0, & |x| \geq th \\
|x|, & |x| < th 
\end{cases}
\]

(4)

5.2 Soft threshold:

In case of soft threshold, if the absolute value of the coefficient is greater than the threshold, then shrinks coefficient in absolute value, otherwise shrinks to zero. The soft threshold function is given by Eqn.5

\[
f(x) = \begin{cases} 
(|x| - \text{sign}(x) \text{Max}(0,|x|-th)), & |x|>th \\
0, & |x| \leq th
\end{cases}
\]

(5)

5.3 Proposed Modified threshold function:

The modified threshold function is represented as shown in Eqn.6

\[
f(x) = \begin{cases} 
-x, & |x| \geq th \\
-x, & |x| < th
\end{cases}
\]

(6)

\( \beta \epsilon (0,1) \), where \( th \) - is threshold and it is given by

\[
th = \sigma_n k \sigma_n
\]

(7)

\( \sigma_n \) is the standard deviation of noise and it is obtained with the help of the robust median estimator from the finest wavelet coefficients (Donoho, 1992) as shown in Eqn.8.
\[ \sigma_n = \frac{\text{median} \left| w_{i,j} \right|}{0.6745} \]  

(8)

\( w_{i,j} \)- Finest or diagonal sub band coefficients of wavelet transform.

To estimate the noise variance \( \sigma_n^2 \) of each curvelet index, the Monte-Carlo simulation method is used (Starck et al., 2000). Few standard white noise images, with zero mean and variance one, i.e., \( N(0,1) \) were discrete curvelet transformed and the variance, \( \sigma_n^2 \) is estimated. \( k \) is a scale-dependent value, \((k=4)\) for the fine scale, \((k=3)\) for detail scales (Starck et al., 2000).

The value of \( \beta \) in Eqn.6 of the modified threshold function lies between \((0-1)\), if its value is \((0)\), the function behaves as hard threshold, if its value is \((1)\), the function behaves as soft threshold. The value of \( \beta \) must have \(0<\beta<1\) to behave Eqn.6 as modified threshold function. For the experimental work the value of \( \beta \) selected as \((0.1)\). The modified threshold function treats non-important coefficients in a different way as compared to hard and soft threshold functions as:

I. The classical threshold function shrinks the coefficients below threshold to zero, but the proposed method shrinks the value of the coefficient to \( \beta \) multiplied to cube of the coefficient divided by the square of the threshold up to a threshold, this avoids discontinuity at threshold and thus overcome the limitation of hard threshold.

II. For absolute value of coefficients greater than threshold, as coefficient value is increasing the deviation between reconstructed and original coefficient is decreasing and thus overcome the limitation of constant deviation as observed in soft threshold.

6. Denoising Procedure:

The steps followed for image denoising based on digital curvelet transform as follows:
I. Find the noise standard deviation \( (\sigma_n) \) Using the equation (8) and from this finding out threshold using equation (7).
II. Take curvelet transform of the input image calculating the curvelet transform coefficients \( c^D(j,k,l) \)
III. Compare the absolute value of coefficients of each sub band of each scale with threshold except first scale.
IV. Change the coefficients as per the modified threshold function using equation (6).
V. Take inverse curvelet transform.

7. Experimental Results and Discussion:

The standard assessment parameters for evaluating Gaussian noise reduction are mean square error (MSE) and peak signal to noise ratio (PSNR).

\[ \text{MSE} = \sum_{r,c} (I_{o}(r,c)-I_{d}(r,c))^2 / R \times C \]  

(9)

\( I_{o}(r,c) \)- Original Image, \( I_{d}(r,c) \)- Denoised image, \( R \) and \( C \) - rows and columns of the image respectively.

MSE indicates average difference of pixels between the original image and the denoise image. MSE should be as low as possible for better result.

\[ \text{PSNR} = 10 \log_{10}(L^2 / \text{MSE}) \]  

(10)

I. Maximum value of pixel in a given image:

PSNR is measured in decibels, it should be large, so that filter removes the noise to a large extent. In order to test the performance of the proposed method, the standard images are corrupted with AWGN (additive white Gaussian noise) and compared with original images. In order to evaluate Gaussian noise removal technique, images are corrupted by Gaussian noise with different variance \((\sigma=20 \text{ and } \sigma=40)\). The images which are used for experimental purpose are shown in Fig.1

![Fig. 1: Images used in experiment a) Mandrill b) Pepper.](image-url)
The proposed method applied to different standard images by adding Gaussian noise with different variance. The efficiency of the method is compared with wavelet hard and soft threshold, curvelet hard and soft threshold and wavelet modified threshold. The Fig. 2 shows the denoised image of mandrill for (σ=40) for different techniques. In Table1 MSE and PSNR are recorded for images mandrill and pepper for different techniques for different values of (σ=20 and σ=40). From the Table1 the proposed method has less MSE and high PSNR as compared to other methods, this is because the proposed method maintains continuity up to the threshold, decreasing the deviation after the threshold as compared to hard and soft threshold respectively. This lower value of MSE shows denoised image is very nearer to the original image and higher value of PSNR indicated denoised image having more information of the original image as compared to other methods. As we observe from the Fig.2, the denoised images by wavelet hard, wavelet soft, wavelet modified, curvelet hard, curvelet soft and proposed method, the visual quality of the proposed method is better as compared to other techniques.

![Denoising of mandrill](image)

**Fig. 2:** Denoising of mandrill a) original image b) image with Gaussian noise c) wavelet hard d) wavelet soft e) wavelet modified f) curvelet hard g) curvelet soft h) proposed method

**Table 1:** Gaussian noise reduction result for mandrill and pepper image.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Metrics</th>
<th>Mandrill</th>
<th>Pepper</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>σ=20</td>
<td>σ=40</td>
<td>σ=20</td>
</tr>
<tr>
<td>Noisy image</td>
<td>MSE 400.4773</td>
<td>PSNR 21.05639</td>
<td>MSE 1597.431</td>
</tr>
<tr>
<td>Wavelet Hard</td>
<td>MSE 148.5078</td>
<td>PSNR 25.36462</td>
<td>MSE 452.3367</td>
</tr>
<tr>
<td>Wavelet soft</td>
<td>MSE 148.5096</td>
<td>PSNR 25.36462</td>
<td>MSE 452.3367</td>
</tr>
<tr>
<td>Wavelet Modified</td>
<td>MSE 143.9133</td>
<td>PSNR 25.50116</td>
<td>MSE 473.7743</td>
</tr>
<tr>
<td>Curvelet hard</td>
<td>MSE 117.6244</td>
<td>PSNR 26.3772</td>
<td>MSE 257.6409</td>
</tr>
<tr>
<td>Curvelet soft</td>
<td>MSE 254.2865</td>
<td>PSNR 23.02894</td>
<td>MSE 419.8752</td>
</tr>
<tr>
<td>Proposed method</td>
<td>MSE 112.8225</td>
<td>PSNR 26.55821</td>
<td>MSE 243.5353</td>
</tr>
</tbody>
</table>

**Conclusion:**

The modified threshold function in curvelet domain is implemented for denoising and it is compared with different techniques as discussed earlier. The assessment parameters PSNR and MSE of the proposed method has high and low value, as compared to other techniques, it shows more noise is removed and preserve the details of the image as compared to other techniques and estimated image is approximately nearer to the original image.

**REFERENCES**


