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Computing Relative weights in AHP and Ranked Units in the Presence of Large Dimensionality of data set based on Orthogonal Gram Schmidt Technique

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ABSTRACT
Background: The purpose of this paper is to determine the local weights in Analytical Hierarchy Process (AHP) by using the maximum variance method based on Gram Schmidt technique in the presence of large dimensionality of data set. This method was utilized for reducing the dimension in data with large dimensionality. In order to gain significant efficiency, the condition should be satisfied (n is the number of Decision Making Units, m is the number of inputs and s is the number of outputs). Since in most presented models in AHP using Data Envelopment Analysis (DEA) experimental principle is not satisfied and have long computing problems, in this paper a new method will be proposed which makes the condition true and reduced the data as well as reducing the dimensions. Also, by reducing the set of performance indicators we will be able to rank decision making units.

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INTRODUCTION

The world around us is full of multi-criterion issues and we have to make decisions constantly. “As discussed by Zhu [21], inputs and outputs weights are usually used in order to calculate this kind of performance measure, and there are two main kinds of approaches to determine the weights: firstly, we could use expert opinions or other managerial information through Delphi or AHP (Analytical Hierarchical Process) method, etc., to specify the weights; secondly, we can use regression or optimization techniques to obtain the weights” [3]. Data envelopment analysis (DEA) methods are of the manners utilized in order to solve such problems. Data envelopment analysis [4,5,7] is a linear programming method for calculating relative efficiency in under evaluation units having various inputs and outputs. Cooper *et al*. [4,7] for the first time introduced DEA for calculating relative efficiency, they stated it as the ratio of weighted output sum to weighted input sum for each decision making unit. The most efficient targets were chosen through diverse way and by combining the decision making priorities in DEA methods.

Analytical hierarchy process (AHP) is a powerful tool for multi-criterion decisions in different contexts. In order to perform (AHP), various techniques exist such as the ones noted in [1,16-18] that each one has its own pros and cons. “Apart from Saaty’s well-known eigenvector method (EM) [12-15], quite a number of alternative approaches have been suggested such as the weighted least-square method (WLSM) [6,10], the logarithmic least squares method (LLSM), the geometric least squares method (GLSM) , the fuzzy programming method (FPM), the gradient Eigen weight method (GEM) , and so on” [19]. “Very recently, Ramanathan [11] developed a DEAHP method for weight derivation and aggregation in the AHP, which views each decision criterion or alternative in a pairwise comparison matrix as a decision making unit (DMU), the row elements of the pairwise comparison matrix as the outputs of the DMUs, and uses a dummy input that has a constant value of one for all the DMUs to build an input oriented CCR model for each DMU” [18].

One of those disadvantages is that the experimental condition $n \geq 3(m+s)$ is not considered and the other one is the required long calculations are in large scales. Since the condition $n \geq 3(m+s)$ does not hold in most of the proposed techniques, in this paper a new method is proposed. It makes the principle true by employing Gram Schmidt orthogonal process for reducing the data as well as for reducing the dimensions.

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There are some studies on reducing the number of input/output variables such as the ones noted in [2,3,5,9]. In this paper first orthogonal Gram Schmidt process is explained and then, employing the stated techniques, the Gram Schmidt variables are calculated and put in the AHP and finally, the relative weights are calculated.

1. Orthogonal Gram Schmidt Process:

Assume that $x_1, x_2, \dots, x_s$ are independent linear vectors. Performing the following procedure, we have:

$$z_1 = x_1$$

$$z_2 = x_2 - \frac{x_2^T z_1}{z_1^T z_1} z_1$$

$$z_3 = x_3 - \frac{x_3^T z_1}{z_1^T z_1} z_1 - \frac{x_3^T z_2}{z_2^T z_2} z_2$$

$$\dots$$

$$z_s = x_s - \sum_{k=1}^{s-1} \frac{x_s^T z_k}{z_k^T z_k} z_k$$

where z_i are independent and orthogonal vectors. For more details see [8]. The orthogonal vectors obtained above are called Schmidt vectors. For example consider the following vectors:

$$x_1 = [1,2,1,0], \; x_2 = [3,3,3,0], \; x_3 = [2,-10,0,0],$$

Since determinant is different from zero, $\det[x_1, x_2, x_3] \neq 0$, so these are linear independent. Performing the above procedure, we have:

$$z_1 = x_1 = [1,2,1,0],$$

$$z_2 = x_2 - \frac{x_2^T z_1}{z_1^T z_1} z_1 = [3,3,3,0] - \frac{[1,2,1,0]^T [3,3,3,0]}{[1,2,1,0]^T [1,2,1,0]}[1,2,1,0] = [1,-1,1,0]$$

$$z_3 = x_3 - \frac{x_3^T z_1}{z_1^T z_1} z_1 - \frac{x_3^T z_2}{z_2^T z_2} z_2 = [2,-10,0,0] - \frac{18}{6}[1,2,1,0] - \frac{13}{3}[1,-1,1,0] = [1,0,-1,0]$$

2. AHP and proposed method based on Gram Schmidt vectors:

Assume that $A = [a_{ii}]_{n\times n}$ is pairwise matrix corresponding to criteria or decision items such that $a_{ii} = 1,\, a_{ij} = 1/a_{ji}$, $(a_{ji} > 0,\, i, j = 1,\dots,n)$. Consider each row of the matrix as a DMU and each component of a DMU as an output. Also assign a constant input with value one to each DMU. Now, by using the CCR multiplicative model in input oriented form and solving the following model for each DMUs $(DMU_o,\, o = 1,\dots,n)$

$$w^* = \max : w_o = \sum_{j=1}^{n} a_{oj} u_j$$

$$s.t.: \quad v_1 = 1$$

$$\sum_{j=1}^{n} a_{ij} u_j - v_1 \leq 0, \quad i = 1,\dots,n$$

$$v_1, u_j \geq 0, \qquad j = 1,\dots,n$$

where $u_j \; (j = 1,\dots,n)$ is j-th weight of output, v_1 weight of input and w^* is the relative weight of the criteria or decision items. Most of methods used the above model for calculating the relative weights in AHP. Now, implement the maximum variance method for paired comparison matrix A.

Step 1: normalize the original variables and name $x_j,\; j = 1,\dots,p$.

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Step 2: set h=1 and choose the variable which has the largest correlation coefficient among the $x_j,\ j=1,\dots,\ p$, and take it as z_1 :

$$\sum_{j=1}^{p} r^2(x_v, x_j) = \underset{i=1,\dots,p}{Max}\left\{\sum_{j=1}^{p} r^2(x_i, x_j)\right\},$$

Step 3: set h=h+1 and calculate the z_{hj} for all $x_j,\ j=2,3,\dots,\ p$ as follows:

$$z_{hj} = x_j - \sum_{k=1}^{h-1} \frac{x_j^T z_k}{z_k^T z_k} z_k$$

Step 4: now, choose h-th gram Schmidt variable, z_h, among $\underset{j=2,\dots,p}{z_{hj}}$ which has the largest variance as the following:

$$\mathrm{var}(z_h) = \underset{j=2,\dots,p}{Max}\left\{\mathrm{var}(z_{hj})\right\}$$

Step 5: repeat step 3 and step 4 until the time all the orthogonal Gram Schmidt variables are obtained.

In order to measure cumulative contribution and to select e variables of all the Gram Schmidt variables, use the following factor. This factor is used to select e variables (e variables among the all Gram Schmidt variables $e \le s$).

$$NP_e = \frac{\sum_{t=1}^{e}\mathrm{var}(z_t)}{\sum_{j=1}^{s}\mathrm{var}(z_j)} \times 100\%$$

where s is the number of all Schmidt variables.

Now, illustrate all Schmidt variables in matrix form $Z=[z_j],\ j=1,\dots,e$. Since these variables should be considered as the output and these may be not positive, hence by using the linear transformation, convert them to positive variables uniformly. Thus, it will become as $r_{pq} = z_{pq} + Q$, where $Q = -\min_{1\le q\le n}(z_{pq}) + 1$.

Now by solving the following model for all e transformed Schmidt variables, relative weights corresponding to each element are obtained.

$$\begin{aligned} &\max w_o = \sum_{j=1}^{e} r_{oj} u_j \\ &s.t.: \\ &\quad \sum_{j=1}^{e} r_{ij} u_j - 1 \le 0, \quad i=1,\dots,n \\ &\quad u_j \ge 0, \quad j=1,\dots,e \end{aligned}$$

For calculating the final weights, the following equation will be employed:

$$W_{A_i}^* = \frac{\sum_{j=1}^{m} w_{ij} w_j}{\max_{k\in\{1,\dots,e\}}\{\sum_{j=1}^{m} w_{ij} w_j\}}, \quad i=1,\dots,n$$

Now, based on the final obtained weights, through the novel presented method, the best choice could be selected.

Note that by using the maximum variance method based on Gram–Schmidt process (present in above) we can reduce data dimensionality. After that by substituting the obtained gram Schmidt variables in one of the ranking models (for example supper efficiency method), we will be able to rank all decision making units [20]. Bian and Li in their paper introduced another ranking method. They considered two characteristics: (1) each decision making unit has a large set of performance indicators; (2) there exist multiple correlations among these indicators.

Conclusion:

In this paper, a new method for reducing data dimensions in calculating weights in analytical hierarchy process for each DMU_s was proposed. First, utilizing orthogonal Gram Schmidt process, maximum variance method was used and based on that from the original variables of the problem of Gram Schmidt variables were obtained. Then Gram Schmidt variables were put in the modified data envelopment analysis and calculated the relative weights in analytical hierarchy process. Performing this procedure, data dimensions are reduced and number of inputs and outputs are reduced and finally the experimental condition $n \geq 3(m+s)$ is satisfied. This method can be used for ranking units as describe in the previous section.

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