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Study of Algebraic Structures (group) in a Fuzzy Finite State Machines

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ABSTRACT

The theory of fuzzy sets has developed in many directions and is finding application in a wide variety of fields. Rosenfeld [1] in 1971 used this concept to develop the theory of fuzzy groups and fuzzy finite state machine introduced by J. N. Mordeson[3] . In this paper, we have given some algebraic concepts such as the concept of fuzzy group on fuzzy finite state machines.

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INTRODUCTION

The theory of fuzzy sets proposed by Zadeh [10] And Wee [7] introduced the concept of fuzzy automata following Zadeh [10,11], fuzzy automata and Fuzzy machine theory has been developed by many researchers. Malik *et al* in [3] introduced the notion of subsystems of a fuzzy finite state machine in order to consider state membership as fuzzy . Several researches were conducted on the generalizations of the notion of fuzzy sets. The study of fuzzy group was started by Rosenfeld [1] and it was extended by Roventa [2] who have introduced the concept of fuzzy groups operating on fuzzy sets. In this paper, we introduce the concept of fuzzy groups with operator on fuzzy finite state machines and. For the sake of convenience, we set out the former concepts.

Preliminaries:

Definition2.1. [10]A fuzzy subset of X is a function from X into [0,1]. The set of all fuzzy subsets of X is called the fuzzy power set of X and is denoted by $FP(X)$.

Definition2,2.[3,6] Let $\mu \in FP(G)$. Then the set $\{\mu(x)|x \in X\}$ is called the image of μ and is denoted by $\mu(x)$ or $Im(\mu)$, The set $\{x \in X|\ \mu(x) \geq 0\}$, is called the support of X and is denoted by suppX. In particular, μ is called a finite fuzzy subset if suppX is a finite set, and an infinite fuzzy subset otherwise. If μ, λ are fuzzy subsets of X, then the fuzzy sets $\mu \cup \lambda, \mu \cap \lambda$ are defined as

$(\mu \cup \lambda)(x) = \mu(x) \vee \lambda(x) = \max\{\mu(x), \lambda(x)\}$

$(\mu \cap \lambda)(x) = \mu(x) \wedge \lambda(x) = \min\{\mu(x), \lambda(x)\}$

Then $\mu \cup \lambda$ and $\mu \cap \lambda$ are called the union and intersection of μ and λ,respectively.

For any collection,$\{\mu_i|i\epsilon I\}$, of fuzzy subsets of X, where I is a nonempty Index set, the least upper bound $\bigcup_{i\in I} \mu_i$ and the greatest lower bound $\bigcap_{i\in I} \mu_i$ of the μ_i's are given by, $\forall x \in X$,

$\bigcup_{i\in I} \mu_i = \bigvee_{i\in I} \mu_i$, $\bigcap_{i\in I} \mu_i = \bigwedge_{i\in I} \mu_i$ respectively.

Definition2.3.[4,6] Let I be a nonempty index set and let $\{X_i|i\epsilon I\}$ be a collection of nonempty sets. Let X denote the Cartesian product of the X_i's, namely ,

$X = \prod_{i\in I} X_i = \{(x_i)_{i\epsilon I}|x_i \in X_i, i \in I\}$

Let $\mu_i \in FP(X_i)$ for all $i \in I$. Define the fuzzy subset μ of X by

$\mu(x) = \bigwedge_{i\in I} \mu_i(x_i)$. $\forall x = (x_i)_{i\in I} \in X$.

Then μ is called the complete direct product of the, μ_i's and is denoted by

$\mu = \prod_{i\in I} \mu_i$. If $I = \{1,2,3, \dots, n\}$, then $X = \prod_{i\in I} X_i = X_1 \times X_2 \times \dots \times X_n$

$= \{(x_1, x_2, \dots, x_n)_{i\epsilon I}|x_i \in X_i, i = 1,2,\dots, n\}$ and we write

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$\prod_{i\in I}\mu_i = \mu_1 \otimes \mu_2 \otimes \ldots \otimes \mu_n$
Clearly, if $\mu_i, \nu_i \in FP(X_i)$ with $\mu_i \subseteq \nu_i$ for all $i \in I$, then $\prod_{i\in I}\mu_i \subseteq \prod_{i\in I}\nu_i$

Some properties on fuzzy groups:

In this paper, G denotes an arbitrary group with a multiplicative binary operation and identity(e). In order to define the notion of a fuzzy subgroup,

Definition 3.1.[6] Let $\mu \in FP(G)$. Then μ is called a fuzzy subgroup of G if
(1) $\mu(xy) \geq \mu(x)\wedge\mu(y)\ \forall x, y \in G$ and
(2) $\mu(x) = \mu(x^{-1})\ \forall x \in G$

Denote by $F(G)$, the set of all fuzzy subgroups of G. If $\mu \in F(G)$, we let
$suppG = \{x \in G|\mu(x) = \mu(e)\}$.
and recall from Definition 3.1 that μ^*denotes the support of μ. If $\mu \in FP(G)$ satisfies condition (1) of Definition 3.1, then $\mu(xn) \geq \mu(x)\ \forall x \in G$
where $n \in$ N. Also, μ satisfies conditions (1) and (2) of Definition 3.1 if and
only if $\mu(xy-1) \geq \mu(x) \wedge \mu(y)\ \forall x, y \in G$.

Lemma 3.2.[1] Let $\mu \in F(G)$. Then $\forall x \in G$,
(1) $\mu(e) \geq \mu(x)$
(2) $\mu(x) = \mu(x^{-1})$

Proof: Let $x \in G$.
(1) $\mu(e) = \mu(xx^{-1}) \geq \mu(x)\wedge\mu(x^{-1}) \geq \mu(x)\wedge\mu(x) = \mu(x)$.
(2) $\mu(x) = \mu((x^{-1})^{-1}) \geq \mu(x^{-1}) \geq \mu(x)$. Hence $\mu(x) = \mu(x^{-1})$.

Fuzzy finite state machine:

Definition 4.1: [8]A fuzzy finite state machine is a triple $M = (Q \times X \times Q)$,where Q and X are finite nonempty sets and μ is a membership of some fuzzy subsets of $Q \times X \times Q$, i.e., $\mu: Q \times X \times Q \to [0,1]$. Let X^* denote the set of all words of elements of X of finite length. Q is called the set of states and X the set of input symbols. Let denote the empty word in X^* and $|x|$ denote the length of x, $\forall x \in X^*$. Define
$\mu^*: Q \times X \times Q \to [0,1]$
$\mu^*(q, \Lambda, p) = \begin{cases} 1 & p = q \\ 0 & p \neq q \end{cases}$
$\mu^*(q, xa, p) = \vee(\mu^*(q, x, r)\wedge\mu(r, a, p)|r\epsilon Q;\ \forall p, q \in Q, x \in X^* . a \in X$

$$\mu^*(q, xy, p) = \vee(\mu^*(q, x, r)\wedge\mu^*(r, y, p)|r\epsilon Q;\ \forall p, q \in Q, x, y \in X^*$$

This means that a fuzzy subset μ of $Q \times X \times Q$ can be naturally extended to a fuzzy subset $\mu^*$of $Q \times X^* \times Q$ under max-min operation.

Finally; if in a fuzzy finite state machine, two sets Q and X consider the two groups with a multiplicative binary operation with identity, then a triple $Q \times X \times Q$ is a group similar group G in the definition 1.3 and by choose an appropriate membership function μ true in definition 1.3 ,Such that μ is a membership of some fuzzy subgroup of $Q \times X \times Q$ i.e $\mu: Q \times X \times Q \to [0,1]$. We can create concept of fuzzy group on a fuzzy finite state machine.

Example: Let $M = (Z_4, Z_2, \mu)$ is a fuzzy finite state machine with two sets of state Z_4 and Input symbols Z_2, which are finite and nonempty sets. Know mathematical systems $(Z_4, +_4)$ and $(Z_2, +_2)$ are finite groups. Now we define $G = (Z_4 \times Z_2 \times Z_4, *)$

To be a finite group, where the operator (*) is define as follows.
$(x * y) = (a_1 +_4 a_2, b_1 +_2 b_2, c_1 +_4 c_2)$
For every $x = (a_1, b_1, c_1), y = (a_2, b_2, c_2) \in Z_4 \times Z_2 \times Z_4$ Now, membership function μ on set $Z_4 \times Z_2 \times Z_4$ are defined as follows,
$\mu: Z_4 \times Z_2 \times Z_4 \to [0,1]$
$\mu(\bar{a}, \bar{x}, \bar{b}) = \begin{cases} \frac{1}{2} & [a] = [b] \\ 0 & [a] \neq [b] \end{cases}$

With membership function defined above. Reference set of fuzzy finite state machine $M = (Z_4, Z_2, \mu)$. Associated with the operation (*) is a fuzzy group because for every two elements x, y of the group $(G, *)$ the following relationship is established.
$\mu(x * y) \geq \mu(x)\wedge\mu(y)$
$\mu(x) = \mu(x^{-1})$

Suppose, for instance $y = (2,0,1)$ and $x = (3,1,2)$, so
$\mu(x * y) = [(3,1,2) * (2,0,1)] = (3 +_4 2, 1 +_2 0, 2 +_4 1) = (1,1,1) = \frac{1}{2}$
On the other $\mu(x) = \mu(3,1,2) = 0$ and $\mu(y) = \mu(2,0,1) = 0$ as a result

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$\mu(x * y) = \frac{1}{2} > \min\{\mu(x), \mu(y)\} = 0$

Conclusion:

In this paper we have considered a new generalization of a fuzzy finite state machine. We have introduced the concept of a fuzzy group in fuzzy finite state machine and investigated several related properties and established a condition for a fuzzy finite state machine to satisfy the fuzzy group exchange property. We also initiated a characterization of a subgroup of fuzzy finite state machine that is fuzzy finite state submachine. Based on these results, we will study retrievable (separated and connected) fuzzy finite state machines, fuzzy transformation semigroups and fuzzy topology associated with a fuzzy finite state machine. And would like to try to find an example of real life problem in respect to philosophical study.

Application: Group theory and fuzzy group theory has many applications in Physics, Chemistry, Genetics, Neurology and Computer science problem.

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