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Forchheimer Equation Coefficients Optimization to Use in Porous Media

Ebrahim Nohani and Ezatolah Alimakan

Department of hydraulic Structures, Dezful Branch, Islamic Azad University, Dezful, Iran

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ABSTRACT

Given that accelerating in cavities of porous media increases the turbulent flow, using Darcy equation and equations that use laminar flow assumptions lack a good reliability. Therefore, relations are required, by considering turbulent flow, to be able to estimate hydraulic gradient with respect to flow velocity and the mechanical properties of the soil. One of the methods of great popularity is Forchheimer binomial equation. In this study, laboratory values are measured by conducting experiments on flow in porous media and for seven soils with different gradation, and Forchheimer coefficients are measured using different methods. The method used in this experiment consisted of the genetic and Particle swarm optimization algorithms. In order to calculate the Forchheimer equation coefficients, initially factors affecting the estimation of the coefficients will be determined using dimensional analysis and then using the dimensionless parameters, the coefficients will be calculated. The results obtaining from the presented methods indicate the superiority of the genetic algorithm compared with Particle swarm optimization algorithm.

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INTRODUCTION

Gravely porous media have a wide application in dam, embankment, coastal development, ripraps and breakwaters. Flow through a rock fill dam, compared to a conventional dam is more impressive. Also, seepage forces on the rock fill will be completely different with force imposed on materials with a non-Darcy flow. Seepage flow through the rock fill changes with flow Reynolds number. Due to turbulent flow in the rock fill, to determine the seepage rate, seepage forces and flow channel in rock fill structure, taking advantage of a reliable non-Darcy relationship is very important. Knowledge of the relationship between velocity and hydraulic gradient is required for engineering design of rock fill structures subjected to cross flow. The subject of this study is finding the best relationship that can well describe the flow characteristics in the samples tested in this study. Over the past few decades, experts and researchers in different fields of science and engineering seek further and better understanding of complicated behavior of non-Darcy flows in porous media. In these years, by conducting numerous studies, including analytic research, experimental research and numerical calculations, researchers have found and suggested various non-linear models that each of these non-linear models has its own strengths and weaknesses. In some models, such as Ergun [6], Wilkins [22], McCorquodale [14] and Martins [12] relations, the coefficients of the seepage flow depends only on the physical parameters of rock fill materials. But in other relations such as Ward [20], the coefficients are not determined only with these parameters and determining of experimental hydraulic conductivity will be required. Due to the high cost of tests, determining the seepage coefficients of the non-Darcy relations in terms of the known parameters of the rock fill is of interest. Different investigators have studied different physical parameters of the porous medium to determine the coefficients of seepage relations and have determined their impact on various experiments with changes in the size, shape and materials [21]. Since the fluid flow in coarse porous media in common applications is a non-Darcy, therefore, linear relations like Laplace equation cannot be used for engineering analysis. In other words, no linear relationship is established between the flow rate (V) and hydraulic gradient (i) in these materials and the ruling relationship will be as exponential or quadratic. Researchers have long been proposed different relations to estimate the hydraulic gradient (i) in terms of mean flow velocity (V) in coarse aggregate that all relationships can be classified in either general forms as follows:

$$i = aV + bV^2 \quad (1)$$

Corresponding Author: Ebrahim Nohani, Department of hydraulic Structures, Dezful Branch, Islamic Azad University, Dezful, Iran.
Tel: +989166148901 E-mail: nohani_e@yahoo.com

Advances in Environmental Biology, 8(21) October 2014, Pages: 475-482

The above binomial known as Forchheimer equation where a and b are coefficients that are a function of the flow characteristics and porous medium, and are usually determined by laboratory tests [19].

$$i = mV^n \quad (2)$$

In above exponential equation: m and n are coefficients dependent on the characteristics of the porous medium and the fluid. Ease of using the exponential equation has led the relationship to be used by different researchers and in various forms [1, 15]. So, it is worth noting that each of the respective researchers considered the determination of m and n coefficients subject to a set of material properties, fluid and flow apparent velocity (or Reynolds number) and so a little consistency exists between the values of the coefficients for a given set of engineering data. On the other hand, despite these differences, there is widespread acceptance that regardless of the method used to determine the coefficients for a given material and in a fairly certain range of the Reynolds numbers, m and n coefficients can be considered as constant with appropriate approximation coefficients [9]. Felton and Herrera [7], Fourar et al. [8], Cheng et al. [5] and Moutsopoulos and Tsihrintzis [15] each of them with a biased perception has presented a model for the non-linear but enduring analysis in coarse porous media. Khalifa et al. [10] studied the effect of the amount of pollution on porous media on permeability coefficient of the material that presented an equation for it based on Hazen formula. Rocha and Cruz [14] published a study in which they have solved the non-compressible fluid seepage three-dimensional flow through porous media by analytical and numeral methods. Balhoff et al. [3] performed a study on a binomial relationship to describe the flow through a porous medium in the range of low Reynolds numbers, which proposed relationship was as an infinite polynomials series in terms of flow rate. Xiexing et al. [23] studied the properties of seepage in sandstone media with various porosity, which result showed that seepage is closely depended on the size of materials and the amount of pressure on it and pores structure. Martins et al. [13] conducted a study on the morphology of cavities and ducts and found that the properties of flow through media with spherical particles changes in accordance with volume of empty space and how it is distributed within the media. Shokri et al. [17] conducted an experimental study on the unsteady flow through gravely porous media; as a result an equation was presented. Azizi et al. [2] studied the effect of porosity on hydraulic gradient in stepped and Gabiony spillways, concluded that porosity greater than the downstream slope affect the pressure drop of flow energy and by reduce the porosity, the energy loss increases. Maleknejad et al. [11] performed a study about the application of Adaptive Neural Fuzzy Inference System in flow hydraulic analysis in a coarse porous media. They conclude that this model due to its smart structures is able to identify the law behind them by calculating the numerical data or examples, without the knowledge of the nature and how they act. Bazargan and Shoaei [4] have conducted a study on the non-Darcy flow analysis in the gravel materials using gradually varied flow theory and which results in new relationships to obtain the hydraulic gradient used in gradually varied flows theories.

Methodology:

Binomial models:

As stated in the introduction, to estimate the hydraulic gradient in terms of velocity, there are two general methods: Forchheimer and exponential relation method and in this study, Forchheimer was used. Forchheimer for the first time in 1901 introduced the relationship between hydraulic gradient and mean flow velocity in the following equation.

$$i = aV + bV^2 \quad (3)$$

In the above equation, $i =$ Hydraulic gradient, $V =$ velocity and a and b are the coefficients of Forchheimer equation. After him, this equation was confirmed by a number of researchers in terms of theoretical credits. They tried in their research to relate a and b parameters to fluid physical properties and porous media and their result has been presented in form of various relationships [18].

Dimensional analysis:

This study aims at presenting the coefficients a and b in Forchheimer equation. Forchheimer in his equations considered these two parameters as constant numbers, while we know that other parameters than velocity are effective in determining the hydraulic gradient that in the study, the effective parameters will be considered to determine the two-parameter a and b. Thus, initially using the dimensional analysis, effective parameters are assumed and after performing the sensitivity analysis and recommending other researchers to use effective parameters, some parameters are directly used in this equation and some show their effect indirectly in other parameters. Effective parameters in determining the hydraulic gradient can be presented as follows:

In the above equation, $i =$ hydraulic gradient, $R =$ hydraulic radius, $V =$ velocity of flow through the channel, $\mu =$ dynamic viscosity of the fluid, $g =$ acceleration of gravity, $k =$ intrinsic permeability coefficient of material, $\rho =$ fluid specific density, $D =$ hydraulic diameter, $f =$ friction coefficient, $n =$ porosity, D_{10}, D_{30} and $D_{60} =$ diameters that are respectively 10, 30 and 60 percent of materials are smaller than them, $C_u =$ Uniformity coefficient, $C_c =$ coefficient of gradation. Dimensionless parameters from above equation can also be presented as follows:

$$i = f(\frac{\rho VD}{\nu} C_u, C_c, n) \quad (4)$$

Parameter $\frac{\rho VD}{\nu}$ is Reynolds number that in open channel flow has no considerable importance and thus to estimate the hydraulic gradient, this parameter will not be considered. According to recommendation of various researchers and on the other hand, our goal is to provide the coefficients of Forchheimer equation, in this study, the following equation is used to estimate the coefficients. This equation is expressed in three different structures and constant coefficients are estimated using the genetic optimization algorithm and Particle swarm optimization.

$$a \,\&\, b = f(\, n,\ C_u, C_c) \quad (5)$$

Defining an objective function is the first step in numerical analysis by which the level of competence of the proposed formulas can be measured. The objective function used in this study is selected as root mean square error which is the accepted and widely used measure in adaptive computing and data processing.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(x_{lab}^{i} - x_{formula}^{i}\right)^2}{n}} \quad (6)$$

In the above equation x lab and x formula are the laboratory data and results obtained from the proposed formulas, respectively. $n =$ Total number of data in this study, considering that we provided seven laboratory data, the value of this parameter was seven. It can be seen that the aforementioned equation calculates errors between the laboratory value and formula result for all seven materials and sums them up and report the result generally. As the objective function, we expect to reduce the value of this error for the proposed formulas. In order to provide different structural equations in this study, due to the nonlinearity of the relationship in equation, different structures will be offered to determine the coefficients a and b. In order to provide these structures with regard to effective parameters obtained using dimensional analysis, different structures in the form of first-order structures (structure I) and second-order (secondary structure) will be presented. Another structure is provided in the form of polynomial which order is estimated based on algorithms presented in this study (genetic and Particle swarm optimization).

RESULTS AND DISCUSSIONS

Primary structural equations:

As the first and simplest structure (in terms of appearance), the following structural equations are proposed for the Forchheimer coefficients.

$$a = X_1 n + X_2 D_{10} + X_3 D_{30} + X_4 D_{60} + X_5 C_u + X_6 C_c \quad (7) \quad b = Y_1 n + Y_2 D_{10} + Y_3 D_{30} + Y_4 D_{60} + Y_5 C_u + Y_6 C_c \quad (8)$$

The above equations are linear combination of independent parameters n,D10, D30,D60,Cu and Cc. Unknowns we are seeking to find their best values or the decision variables in the optimization process are twelve parameters X1, X2 ...Y6. At this stage, the genetic algorithm and Particle swarm optimization method are used to obtain these unknown coefficients according to the objective function. To do so, the program written in BASIC is used. Table 1 provides the results from evolutionary algorithms in this study (Genetic Algorithm and Particle swarm optimization) to provide a and b coefficients that are given in the form of equations. According to the table it can be seen that in parameter a, except the coefficients X1 and X5, the others are zero. In parameter b, coefficients Y2 and Y6 have values. So according to the values presented in the tables and equations, while maintaining the overall shape of the Forchheimer equation, to calculate the hydraulic gradient, the following equations can be written for both genetic algorithm and Particle swarm optimization as follows:

Genetic Algorithms:

$$a = 79.6n + 2.25C_u \quad (9)$$

$$b = 0.077D_{10} + 0.364\ C_c \quad (10)$$

So Forchheimer equation can be presented as follows:

$$i = (79.6n + 2.25C_u)V + (0.077D_{10} + 0.364C_c)V^2 \quad (11)$$

Particle swarm optimization Algorithm:

$$a = 79.18n + 2.295C_u \quad (12)$$

$$b = 0.076D_{10} + 0.361C_c \quad (13)$$

$$i = (79.18n + 2.295C_u)V + (0.076D_{10} + 0.361C_c)V^2 \quad (14)$$

It can be seen that the results provided by both algorithms are almost the same, as in both equations, coefficients a and b in each of the algorithms depends on the same parameters and coefficients Xi and Yi provided for each one of them is relatively equal. With obtaining the unknown values by using genetic algorithm and Particle swarm optimization, the proposed equations of first structure can be used to calculate the Forchheimer coefficients and compare the results with laboratory data. Table 2 shows the coefficients obtained using genetic algorithms and Particle swarm optimization for all 7 soils. Therefore, using the Forchheimer equation i = av + bV2 and using the table below for each of the proposed aggregation, the modified equation of Forchheimer can be presented to calculate the hydraulic gradient.

Figure 1 shows the results of estimating first coefficient of Forchheimer equation (a) for various soils at first structure. According to figure, it can be seen that the results provided by the use of Particle swarm optimization and genetic algorithm are the same for almost all soils. It can be also seen that except for the soil type 1 and type 2, the predicted values using these algorithms are in good agreement with the laboratory values.

Table 1: Optimized values of unknown parameters of first proposed structure equations.

		Genetic Algorithm (GA)	Particle swarm optimization (PSO)			Genetic Algorithm (GA)	Particle swarm optimization (PSO)
Coefficient a	X1	79.579	79.117	Coefficient b	Y1	0	0
	X2	0	0		Y2	0.0767	0.0760
	X3	0	0		Y3	0	0
	X4	0	0		Y4	0	0
	X5	2.249	2.295		Y5	0	0
	X6	0	0		Y6	0.3643	0.3609
	RMSE error value	1205.022	1210.694		RMSE error value	1.796	1.808

Table 2: Calculation of Forchheimer coefficients values by optimized parameters.

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational value of a (GA)	41.35	32.08	36.95	32.62	36.84	36.78	36.90
Computational value of a (Particle swarm optimization)	41.21	31.98	36.86	32.51	36.70	36.67	36.80
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (GA)	0.7897	1.3476	0.8208	1.0277	1.1228	0.7923	1.0126
Computational value of b (Particle swarm optimization)	0.7823	1.3348	0.8130	1.0179	1.1122	0.7848	1.0030

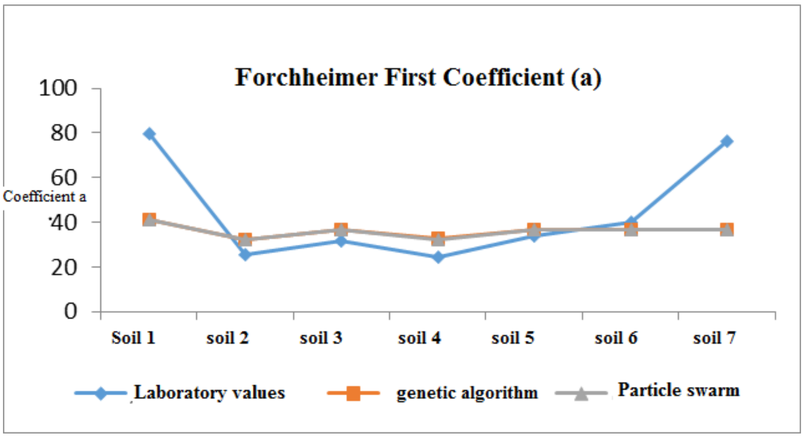

Fig. 1: comparison of the results of the first structure formulas for first Forchheimer coefficients.

Figure 2 shows the results of the estimation of second coefficient of Forchheimer equation (b) for the first structure using the methods presented in this study (Genetic Algorithm and Particle swarm optimization Algorithm), and results from laboratory study. According to this figure, it can be seen that the estimates enjoy a good degree of accuracy, so that estimated values and the values of the laboratory results are almost equal except for the soil 4. Also in this figure, it can be seen that both the proposed algorithms, although sometimes results of genetic algorithms has very little difference with Particle swarm optimization algorithm, have better accuracy.

Advances in Environmental Biology, 8(21) October 2014, Pages: 475-482

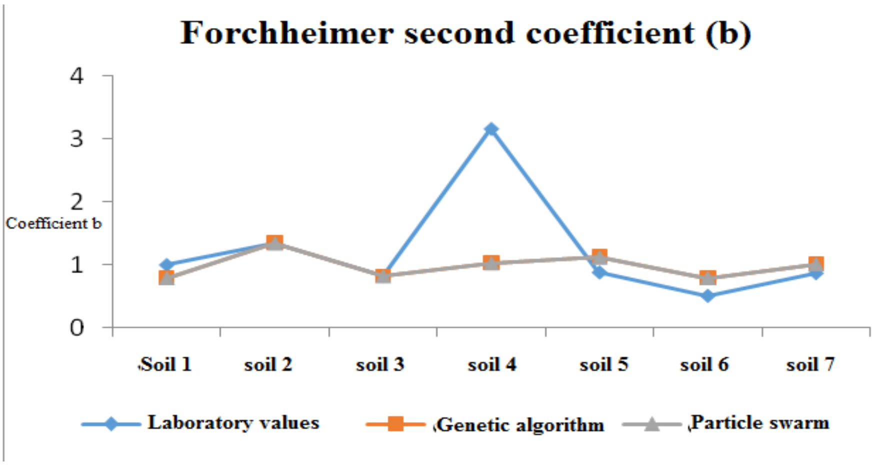

Fig. 2: Comparison of the results of first structure formulas for the second Forchheimer coefficients

Secondary structural equations:

As the secondary structural equations, we tested the binary for the input properties. This structure has a shape similar to the first except that the effective parameters in the estimation of the coefficients a and b are presented in order two. The secondary structure equations are defined as following.

$$a = X_1 n^2 + X_2 D_{10}{}^2 + X_3 D_{30}{}^2 + X_4 D_{60}{}^2 + X_5 C_u{}^2 + X_6 C_c{}^2 \tag{15}$$

$$b = Y_1 n^2 + Y_2 D_{10}{}^2 + Y_3 D_{30}{}^2 + Y_4 D_{60}{}^2 + Y_5 C_u{}^2 + Y_6 C_c{}^2 \tag{16}$$

Table 3 shows the results of the genetic algorithm and Particle swarm optimization. The purpose of using these two algorithms is calculating the coefficients a and b that are presented as relations. According to the table, it is observed that for parameter a, coefficients X1 and X5, that are parameters of n and Cu respectively and for parameter b, coefficients Y2, Y5 and Y6 have values. So according to the values presented in the tables and relations, while maintaining the overall shape of the Forchheimer equation, to calculate the hydraulic gradient, the following equations can be written for both genetic algorithm and Particle swarm optimization as follows:

Genetic Algorithms:

$$a = 185.8n + 0.379\ C_u \tag{17}$$

$$b = 0.0062 D_{10} + 0.87 C_u + 0.148\ C_c \tag{18}$$

So Forchheimer equation is presented as follows:

$$i = (185.8n + 0.379 C_u)\ V + (0.0062 D_{10} + 0.87 C_u + 0.148 C_c)\ V^2 \tag{19}$$

Particle swarm optimization Algorithm:

$$a = 186.02n + 0.407 C_u \tag{20}$$

$$b = 0.007 D_{10} + 0.82 C_u + 0.14 C_c \tag{21}$$

$$i = (186.02n + 0.407 C_u)\ V + (0.007 D_{10} + 0.82 C_u + 0.14 C_c)\ V^2 \tag{22}$$

It can be seen that the results provided by both algorithms are almost the same, so in both equations, the coefficients a and b are both dependent on similar parameters in each two algorithms and coefficients Xi and Yi presented for each of them are relatively equal. Table 4 presents the parameters a and b that are Forchheimer equation coefficients and one of the main objectives of this study for different soils according to the mechanical properties and soil aggregation. Therefore, using the Forchheimer equation form and the following table, the following equations are presented to calculate the hydraulic gradient for each aggregation offered. Figure 3 shows the results of estimating the Forchheimer equation coefficient (a) that are presented as secondary structural equations and are calculated in this study using genetic algorithm and Particle swarm optimization algorithms. According to figure, it can be seen that the results of estimates done by these two algorithms has a procedure like first structure. This figure shows that almost the estimations done by both algorithms are identical and for soils 1 and 7, it has not enjoyed a fairly good estimation while for other soils the values estimated have a good adaption with the actual values. Figure (4) shows the results of the estimation of Forchheimer second coefficient (a) for seven soils with different particle sizes. In this figure, estimations are presented by using genetic algorithms and particle swarm optimization. According to the figure, results of the estimation of these coefficients are identical for all soils by algorithms and the results provided by both are the same.

Advances in Environmental Biology, 8(21) October 2014, Pages: 475-482

Table 3: Optimized values of the unknown parameters of 2nd proposed structural equations.

		Genetic Algorithms (GA)	Particle swarm optimization (PSO)			Genetic Algorithms (GA)	Particle swarm optimization (PSO)
Coefficient a	X1	185.799	186.019	Coefficient b	Y1	0	0
	X2	0	0		Y2	0.0062	0.0070
	X3	0	0		Y3	0	0
	X4	0	0		Y4	0	0
	X5	0.379	0.407		Y5	0.087322	0.081655
	X6	0	0		Y6	0.1481	0.1401
	RMSE error value	1264.765	2659.269		RMSE error value	3.856	3.904

Table 4: Calculating the values of Forchheimer coefficients by optimized parameters.

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational value of a (GA)	42.60	25.34	31.78	26.29	34.62	32.73	32.60
Computational value a (Particle swarm optimization)	42.72	25.43	31.95	26.37	34.70	32.86	32.74
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (GA)	0.3225	0.3436	0.3616	0.3378	0.3219	0.3220	0.3799
Computational value of b (Particle swarm optimization)	0.3099	0.3383	0.3475	0.3272	0.3142	0.3099	0.3664

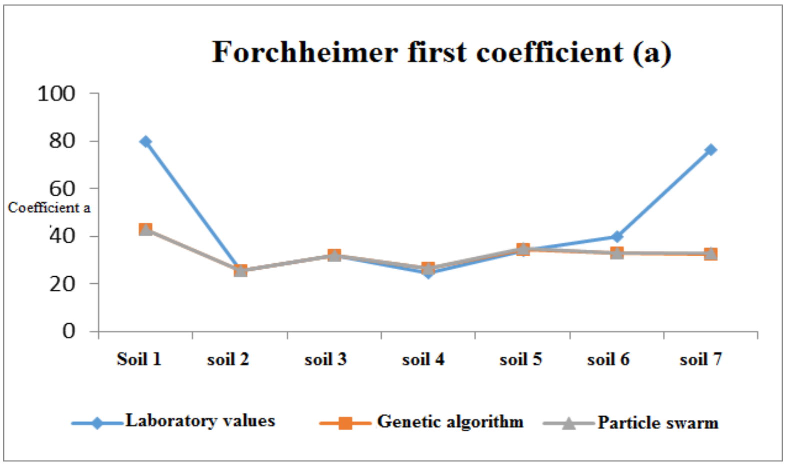

Fig. 3: Comparison of the results of the secondary structure formula for Forchheimer first coefficient.

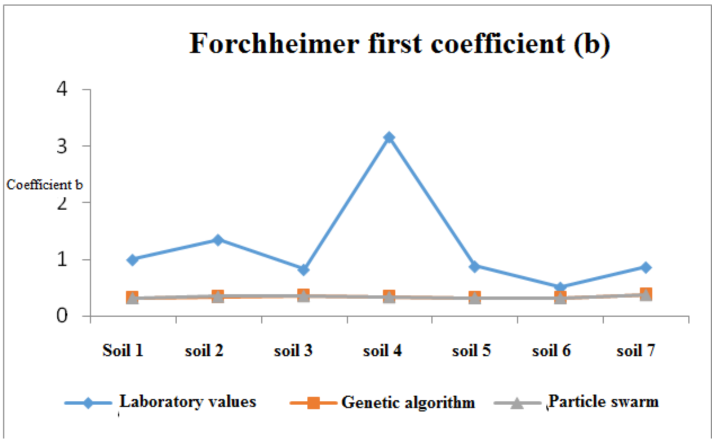

Fig. 4: comparison of the results of second structure formula for Forchheimer second coefficient.

Tertiary structural equations:

To define the tertiary structural equations, we see the results of two previous structures. It can be seen that about the Forchheimer first coefficient i.e. coefficients a, in both previous structures only the parameters n (unfilled) and Cu (form factor) effect the equation and other parameters are zero. Therefore, in new proposed structure for coefficient a, only these two quantities are used and in this stage, both coefficient and exponent are considered as unknown. In addition, to allow optimization algorithms to achieve a better fit, the product of the quantity is included in the new structure. By the same reasoning, a new structure will be defined for Forchheimer second coefficient i.e., coefficient b.

$$a = X_1 n^{X_2} + X_3 C_u^{X_4} + X_5 n C_u \quad (23)$$

$$b = Y_1 D_{10}^{Y_2} + Y_3 C_c^{Y_4} \quad (24)$$

The results of the optimization process of the equation are as follows.

Figure 5 compares the results of the estimation of Forchheimer first coefficient for seven different soils with laboratory results. According to figure, it can be seen that accuracy of estimations done in structure has a better precision than primary and secondary structures and good results are provided for almost all soils and in most of soils (1, 2, 4, 5, 6) the estimated and laboratory values observed have a good fit. In this structure like two first structures, the results of both algorithms used are the same.

Figure 6 compares the results of estimating the Forchheimer second coefficient by the tertiary structural equations provided in this study and the genetic evolutionary algorithms and particle swarm optimization with the lab results. According to the figure, it can be seen that except for soil No. 4, for almost all soils, the results estimated using both algorithms is almost identical with the observed data. In this structure, as well as the primary and Secondary, it can be seen that both algorithms give the same results.

Table 5: Optimized values of the unknown parameters of third proposed structure equations.

		Genetic Algorithms (GA)	Particle swarm optimization (PSO)			Genetic Algorithms (GA)	Particle swarm optimization (PSO)
Coefficient a	X1	10220420	10228549	Coefficient b	Y1	0.0768	0.076
	X2	16.56424	16.56019		Y2	0.9910	0.9880
	X3	1.258538	1.258109		Y3	0.3654	0.3599
	X4	0.5018282	0.5019981		Y4	1.008	0.9807
	X5	46.408	46.890		Y5		
					Y6		
	RMSE error value	527.784	523.138		RMSE error value	1.806	1.831

Table 6: Calculation of values of Forchheimer coefficients by optimized parameters.

	Soil 1	Soil 2	Soil 3	Soil 4	Soil 5	Soil 6	Soil 7
Laboratory values of coefficient a	79.89	25.34	31.78	24.34	34.06	39.79	76.21
Computational values of a (Genetic Algorithm)	79.87	25.42	46.28	25.90	34.00	39.87	41.80
Computational value of a (Particle swarm optimization)	80.41	25.67	46.72	26.15	34.29	40.24	42.20
Laboratory values of coefficient b	0.9946	1.347	0.82	3.159	0.8758	0.504	0.8631
Computational value of b (Genetic Algorithm)	0.7845	1.3270	0.8132	1.0188	1.1099	0.7858	1.0038
Computational value of b (Particle swarm optimization)	0.7727	1.3044	0.8018	1.0010	1.0912	0.7747	0.9865

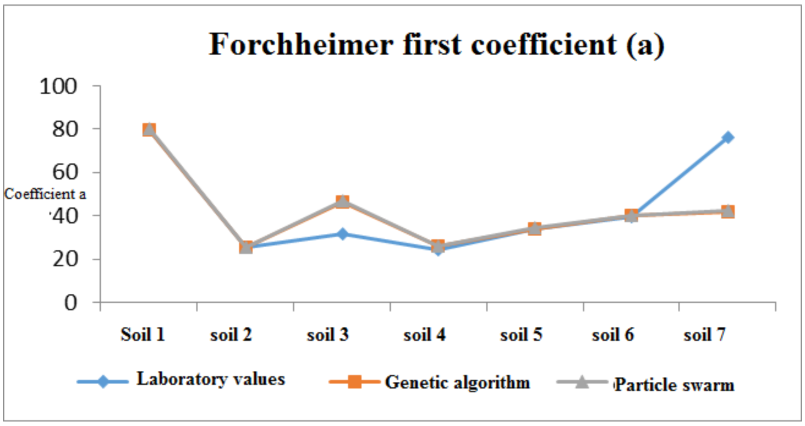

Fig. 5: Comparison of the results of the tertiary structure formulas for Forchheimer first coefficients.

Advances in Environmental Biology, 8(21) October 2014, Pages: 475-482

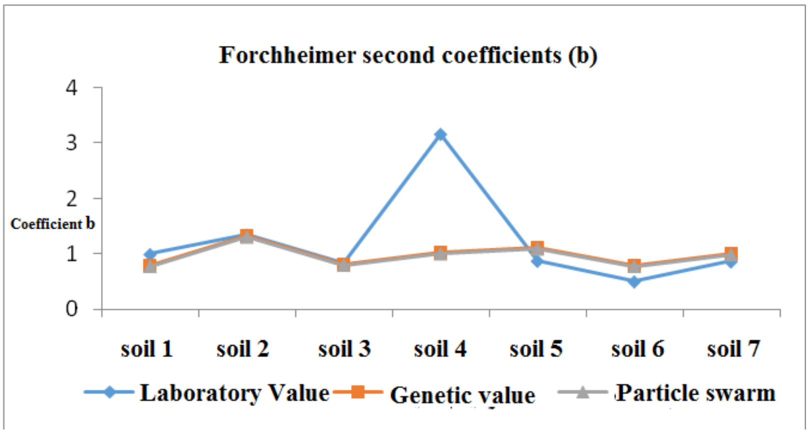

Fig. 6: Comparison of the results of tertiary structure formulas for Forchheimer second coefficients

Conclusion:

According to the presented results, the best relationships presented for the Forchheimer coefficients can be selected in this manner that the first coefficient of Forchheimer equation (a) can be calculated using tertiary structural equation and second coefficient of the equation (b) using the primary structural equation presented in this study. These structural equations are able to predict the values of first and second coefficients of Forchheimer equation in terms of mechanical properties of materials. The selected equations are presented as follows:

$$a = 10220420 n^{16.56} + 1.259 C_u^{0.501} + 46.408 n C_u \quad (25)$$

$$b = 0.0767\ D_{10} + 0.3643 C_c \quad (26)$$

The results obtained from coefficients of Forchheimer equation indicates that both genetic algorithm and particle swarm optimization have relatively the same accuracy, although the results offered by the genetic algorithm have partially more accuracy in predicting coefficients.

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