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A Novel Combinatorial Method of DEA and Artificial Neural Network for Estimating Returns to Scale and Benchmarks

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ABSTRACT

In today's banking industry the efficiency and quality of services is received more attention due to improvement of technology. The competition in this sector has become increasingly intense. Hence, the performance measurement and strategic planning is necessary. The leading approach for efficiency measurement and benchmarks is Data envelopment analysis. As data envelopment analysis could not forecast the benchmarks for future, for the first time in literature artificial neural network is used as a complementary method in this paper to measure returns to scale and most productive scale size benchmarks in five years. Hundred branches of one of the Iranian commercial banks were selected as a case study in this research.

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INTRODUCTION

Banks have become cornerstones of our economy for several reasons. Undoubtedly the role of financial institutions in our lives is undeniable. Banks are among the hardest to analyze. Various studies such as Berger and Humphrey (1997) tried to analyze banks. Data envelopment analysis (DEA) is the most used method for measuring efficiency, returns to scale, and benchmarks.

The try to realize the relationship between the technology of a decision-making unit, its efficiency and environment, and the measurement of returns to scale (RTS) is not new to DEA. The concept of returns to scale (RTS) has been widely studied during the development period within different frameworks. Banker [1] was first introduced the standard model for analyzing returns to scale in DEA. Continuing in this tradition, the important new concept of MPSS (most productive scale size) was introduced by Banker [2].

The economic concept of returns to scale (RTS) has been widely studied within the different frameworks during this period of model development. RTS is the topic, which this paper devoted to. Two major paths were exists in literature in treating returns (RTS) in DEA. First path, which is mentioned by Fare, Grosskopf and Lovell [9], determines RTS by concept of ratios of radial measures. Second path, that is the one we follow, includes, but is not restricted to, radial measure models and introduced by Banker [2], Banker *et al*. [2], Banker and Thrall [5] and Banker and Maindiratta [4].

Seiford and Zhu [11] provided a simple approach to RTS estimation without the need for checking multiple optimal solutions by establishing the relations among the alternative approaches. Using the advantages of the nonlinear nature of Artificial Neural Networks (ANNs), Vouldis *et al*. [13] and Michaelides *et al*. [10] combined ANN and DEA to deal with the endogeneity of outputs. But there are almost no studies in literature, which forecasted RTS. In this paper artificial neural network was selected as an auxiliary method to forecast RTS in five years. Then the concept of the most productive scale size (MPSS) was selected to treat a multiple output-multiple input case without departing from returns to scale concepts built around the single output case in classic economic models. And projections on MPSS were reported. The paper is arranged as follow. In Section 2, a brief description of artificial neural network, returns to scale, and projection can be seen. Section 3 defines models and methodology used in this paper. The results are given in Section 4. At last, Section 5 includes conclusions and future works.

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2. Problem definition:

2.1 Artificial neural network:

Artificial neural networks were inspired of neural networks mainly workings of the brain. Novel structure of the information-processing system is the main part of this paradigm. In such systems large number of highly interconnected processing neurons working together to solve specific problems. Similar to people artificial neural networks (ANNs) learn by examples and trained by adjusting the weights between neurons so that an input leads to a target output.

Over the last decade various applications of ANN were introduced, one of the major applications of ANN in business application is in forecasting (Sharda, 1994). Many ANN models have been proposed since 1980s such as, Multi-layer perceptron (MLP), Hopfield networks, and Kohonen's self-organizing networks, which are the most influential models.

Due to inherent capability of arbitrary input – output mapping, the MLP networks are used in several problems especially in forecasting. As it shown in Fig.1, MLP networks include several layers of nodes. The input layer is the lowest layer, where the information received. The layer in which result displays is output layer.

Input layers and output layers are connected with intermediate layers, which named hidden layers. Nodes in adjacent layers, are connected by acyclic arcs from a lower layer to a higher layer.

Acyclic arcs from a lower layer to a higher layer connected the nodes in adjacent layers.

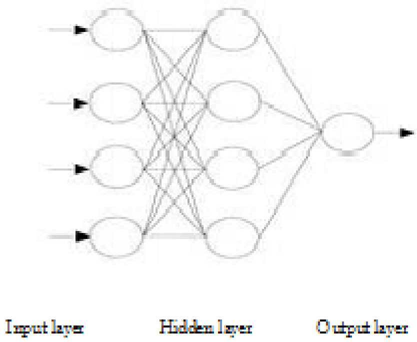

Fig. 1: The structure of three layers MLP network.

Mostly multilayer network trained using the backpropagation (BP) algorithm for forecasting, which are a class of feed-forward neural networks. BP neural networks consist of a collection of inputs and processing units named as neurons and the direction of information flow is from the input to the output layer, with supervised learning rules. In the learning process the known correct answer is compared with each network's forecast and the weights are adjusted based on minimizing the error function. For instance for forecasting the value of $x(t+1)$ in time series like $x(1)...x(t)$, $x(t-k+1)..x(t)$ is chosen as the inputs to multilayer network and the output will be the forecast. The network uses data that extracted from the historical time series for training and testing on large training and testing sets.

ANN must be trained to perform any task. Through the training process, arc weights, which are the key factors of an ANN, will be demonstrated.

Learned knowledge saved in arcs and nodes in the form of arc weights and node biases. The MLP training is a method of training, in which the desired response of the network (target value) for each input pattern (example) is always available.

The steps in training process are usually as following. Firstly, examples of the training set are entered into the input nodes. Secondly, the activation values of the input nodes are weighted and accumulated at each node in the first hidden layer. Lastly, activation value is obtained by an activation function, which is transforming the total into activation value. The value becomes an input into the nodes in the next layer. This process works until the output activation values are found. The training algorithm is trying to the weights that minimize the mean squared errors (MSE) or the sum of squared errors (SSE).

2.2 Returns to scale:

The basic model for analyzing returns to scale in DEA firstly addressed explicitly in the Harvard Business School thesis of Banker [1] in which it was formulated in terms of dual pair of linear programming problems.

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$$Min\ \theta_0 - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

Subject to

$$0 = \theta_0 - \sum_{i=1}^{n} x_{ij}\lambda_j - s_i^-, \forall i$$

$$y_{ro} = \sum_{j=1}^{n} \lambda_j - s_r^+, \forall r$$

$$1 = \sum_{j=1}^{n} \lambda_j, \quad 0 \le \lambda_j, s_i^-, s_r^+, \forall i, \forall r, \forall j \tag{1}$$

The dual (multiplier) model of the (BCC) method mentioned above in (1) is obtained from the same data that are then used in the following form:

$$Max \quad \sum_{r=1}^{s} \mu_r y_{r0} + u_0$$

Subject to

$$-\sum_{i=1}^{m} \nu_i x_{ij} + \sum_{r=1}^{s} \mu_r y_{rj} + u_0 \le 0, \forall j$$

$$\sum_{i=1}^{m} \nu_i x_{io} = 1,$$

$$-\nu_i \le -\varepsilon, \forall i$$

$$-\mu_r \le -\varepsilon, \forall r \tag{2}$$

Where:

x_{ij} = Observed amount of input i for DMU_j,

y_{rj} = Observed amount of output r for DMU_j,

With i=1... M; r=l... s; j=1, n; and DMU_0 is one of the DMU_j(= Decision Making Units) whose input-output record is to be evaluated for its efficiency based on the performance of all DMU_j (including itself).

According to Banker [1], the focus of the returns to scale analysis was on the sign of u_0.

Banker, Charnes and Cooper [2] described this where unique solutions were explicitly assumed. Along with this tradition, Banker [2] introduced the important new concept of MPSS (most productive scale size). MPSS happens when (xX_0, yY_0) with $0 \le x \le y$ for augmentations of output in association with input augmentation in the neighborhood of (X_0, Y_0) in which X_0 and Y_0 are the input and output vectors for DMU_0 while x and y are scalars. The above statements with respect to returns to scale in DEA are all precisely described in Banker and Thrall (1992), who also extend the analysis to allow for alternate optima, in which they give the following characterizations:

a. Increasing returns to scale conquer at (X_0, Y_0) if and only if $u_0^* > 0$ for all optimal solutions.

b. Decreasing returns to scale conquer at (X_0, Y_0) if and only if $u_0^* < 0$ for all optimal solutions.

c. Constant returns to scale conquers at (X_0, Y_0) if and only if $u_0^* = 0$ in any optimal solution.

A solution with a new $u_0 = 0$ means that returns to scale is locally constant. Otherwise all alternate optima have the sign originally obtained from (1)-(2), and all possibilities are thus covered as required in characterizations a, b, and c.

It may be remarked, (X_0, Y_0) are the coordinates of the point on the efficiency frontier, which are obtained from projection on BCC frontier in the evaluation of DMU_O by the solution to (1). Hence, that a use of the projection makes it unnecessary to assume that the points to be analyzed are all on the BCC efficient frontier same as what was assumed in Banker and Thrall [5]. An examination of all optimal solutions can be time consuming. Hence, Banker and Thrall [5] provide a way of avoiding a need for examining all optimal solutions.

Although, Banker *et al*. [3] approach is used here because it avoids the possibility of infinite solutions that are present in the Banker–Thrall approach. Moreover, the Banker *et al*. [3] approach insures that the RTS analyses are conducted on the efficiency frontier. This is accomplished by (3). Assume that an optimum has been achieved with $u_0^* < 0$. According to Banker *et al*. [3], the following model may then be used to avoid having to explore all alternate optima,

$$Max\ \hat{u}_0$$
$$Subject\ to$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - \hat{u}_0 \le 0,\ j = 1,\ldots,n;\ j \ne 0,$$
$$\sum_{r=1}^{s} u_r \hat{y}_{r0} - \sum_{i=1}^{m} v_i \hat{x}_{i0} - \hat{u}_0 \le 0,$$
$$\sum_{i=1}^{m} v_i \hat{x}_{i0} = 1,$$
$$\sum_{r=1}^{s} u_r \hat{y}_{r0} - \hat{u}_0 = 1,$$
$$v_i, u_r \ge 0\ \ and\ \ \hat{u}_0 \le 0, \qquad (3)$$

If $\hat{u}_0^* = 0$ can be obtained then condition (c) is satisfied and returns to scale are constant. If $\hat{u}_0^* < 0$, then returns to scale are increasing. In this way of implementing the need for examining all alternate optima is avoided. If $u_0^* > 0$ then the modified version of model (3) must be solved, the objective function will be $Min\ \hat{u}_0$ and $\hat{u}_0 \le 0$ will alter to $\hat{u}_0 \ge 0$. In this case, if $\hat{u}_0^* > 0$, then returns to scale are decreasing, else the returns to scale are constant.

2.3 Projections:

Returns to scale has an unambiguous concept only if the point (X_0, Y_0) is on the efficiency frontier. As recommended by Banker and Thrall [5] for bounding the scale elasticity, it is assumed that (X_0, Y_0) is on the efficiency frontier but we can omit the need for this assumption by using the following projections as suggested in Charnes, Cooper and Rhodes (1978). It means there is no need to be concerned about the efficiency status in analyses because efficiency can always be achieved as follow, we can use optimal values from (1) to mirror this DMU_0 onto the BCC efficiency frontier via the following formulas, If a DMU_0 is not BCC efficient:

$$\hat{x}_{i0} = \theta_0^* x_{i0} - s_i^{-*},\ i = 1,\ldots,m,$$
$$\hat{y}_{r0} = y_{r0} + s_r^{+*},\ r = 1,\ldots,s,$$

In which, $\hat{x}_{i0}$, $\hat{y}_{r0}$ are new values for the corresponding inputs and outputs with $\hat{x}_{i0} \le x_{i0}$ and $\hat{y}_{r0} \ge y_{r0}$ as reached from optimal values for the variables $\theta_0^*, s_i^{-*}, s_r^{+*}$. The symbol "*" mentioned an optimal value.

The projection on MPSS calculated by means of is following formulas:

$$(X,Y) = \left(\frac{\theta^* x_0 - s_i^{-*}}{1/\ ^*}, \frac{y_0 + s_r^{+*}}{1/\ ^*} \right)$$

3. ANN-MPSS:

In this research ANN is applied to forecast the input and outputs of each DMU in five years. After preliminary analyses and trial, Levenberg–Marquardt algorithm, the fastest training algorithm is chosen for proposed MLP network. Levenberg–Marquardt algorithm, can be considered as a trust-region modification of the Gauss–Newton algorithm. Fig. 2. Shows the two samples of test and train charts for proposed ANN.

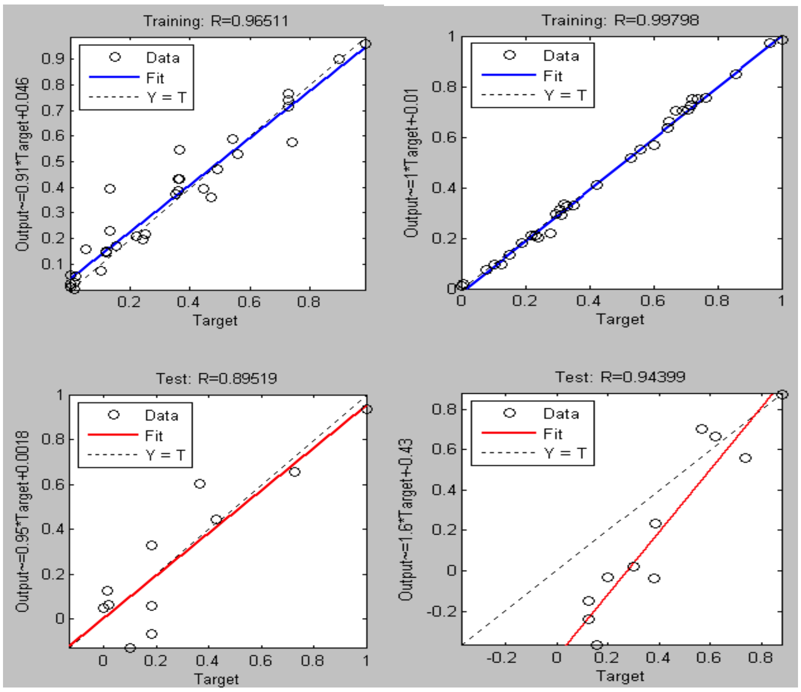

Fig. 2: Training and testing charts.

Fig.2. displays the good quality of the trained network prediction. After forecasting inputs and outputs by ANN, the BCC model was selected for estimating returns to scale and finally, MPSS benchmarks were calculated.

4. Computational results:

Twenty branch of one of Iranian commercial banks were selected and the related data were collected. The data cover the period of March to February in year 2006 to 2011. Each branch demonstrates a decision-making unit (DMU) and uses two inputs to produce seven outputs as it shown in Table 1.

Table 1: Inputs and outputs of branches

Outputs	*Inputs*
1- Income condominium (Y_1)	1- Deposit's paid profit (X_1)
2- Fee (Commission) (Y_2)	2- Expenses (personnel & official) (X_2)
3- Other income (Y_3)	
4- Main deposits (Y_4)	
5- Other deposits (Y_5)	
6- Current deposit (Y_6)	
7- Loan granted account (Y_7)	

Table 2: displays efficient DMU's returns to scale by BCC model.

	Efficiency	$u_0^*(1)$	$u_0^*(2)$	$u_0^*(3)$	Returns to scale
DMU01	1	-1	0	-1	Decreasing
DMU02	1	22.3	-46.58	0	Constant
DMU03	1	5.23	0.66	-7.96	Increasing
DMU04	0.72	8.76	-0.72	-0.73	Inefficient
DMU05	1	-0.06	3.86	-0.27	Decreasing
DMU06	1	0	0	-0.48	Constant
DMU07	1	17.55	146.9	-17.86	Increasing
DMU08	1	-10.80	387.8	1.00	Constant
DMU09	0.77	-0.18	156.3	0.20	Inefficient
DMU10	0.83	-2.27	-2.27	-31.81	Inefficient
DMU11	0.87	2.22	-0.34	0.13	Inefficient

DMU12	1	-1	0	-1	Decreasing
DMU13	1	0	694.07	-0.23	Constant
DMU14	1	2.80	9.63	-2.21	Increasing
DMU15	0.71	-1.41	3.67	-2.62	Inefficient
DMU16	1	-1.82	69.08	-2.73	Decreasing
DMU17	1	-0.13	0.98	-1	Decreasing
DMU18	1	-38.43	0.58	0.96	Constant
DMU19	0.99	0.81	7.98	0.46	Inefficient
DMU20	1	0	542.95	-3.39	Constant

Benchmarks on MPSS were shown in Table 3.

Table 3: Projection of each DMU on MPSS.

	$x1^{*}$	$x2^{*}$	$y1^{*}$	$y2^{*}$	$y3^{*}$	$y4^{*}$	$y5^{*}$	$y6^{*}$	$y7^{*}$
DMU01	-6.63E+10	4.28E+07	2.16E+08	-1.85E+07	2.73E+10	6.96E+10	2.55E+09	4.14E+08	1.17E+10
DMU02	6.09E+08	1.37E+08	1.16E+09	1.95E+10	9.65E+10	1.79E+11	1.45E+10	4.69E+09	1.92E+11
DMU03	2.84E+08	6.06E+07	1.35E+09	1.30E+10	1.30E+10	6.32E+10	2.60E+09	1.98E+10	5.76E+10
DMU04	1.04E+11	4.21E+07	3.89E+08	2.19E+09	1.82E+10	1.86E+11	4.12E+09	2.43E+09	-1.85E+11
DMU05	3.58E+09	4.04E+07	1.56E+09	9.87E+09	6.02E+10	9.74E+10	4.83E+09	3.29E+10	1.08E+11
DMU06	2.13E+09	3.81E+07	-8.41E+09	4.49E+07	2.45E+10	1.54E+11	1.05E+10	5.99E+07	2.01E+09
DMU07	1.84E+07	-3.29E+06	3.36E+08	2.42E+06	3.32E+10	1.13E+10	2.49E+08	3.26E+09	2.05E+10
DMU08	6.58E+07	4.56E+07	1.77E+08	2.43E+07	2.33E+10	5.89E+10	2.79E+08	4.00E+10	1.09E+10
DMU09	2.29E+09	3.82E+07	7.97E+08	2.31E+09	3.25E+10	4.81E+10	2.12E+08	4.80E+10	3.12E+10
DMU10	1.30E+08	8.88E+06	6.85E+07	1.40E+06	1.90E+10	3.54E+10	1.54E+08	3.70E+10	5.69E+09
DMU11	5.80E+07	5.58E+06	2.38E+08	3.42E+06	3.68E+10	2.57E+10	2.94E+07	2.01E+10	1.43E+10
DMU12	3.35E+07	-9.83E+06	5.64E+07	8.53E+07	1.65E+10	2.05E+10	-1.53E+09	4.65E+10	2.84E+09
DMU13	2.22E+08	1.14E+07	1.23E+09	1.23E+09	9.38E+09	2.37E+10	2.98E+09	5.61E+04	-1.44E+09
DMU14	2.02E+08	1.15E+07	9.32E+07	2.51E+07	-4.75E+10	5.32E+10	3.08E+09	4.29E+06	5.82E+09
DMU15	4.03E+07	1.50E+07	1.03E+08	6.14E+06	2.00E+10	4.07E+10	3.69E+08	3.61E+10	5.87E+09
DMU16	1.01E+08	1.01E+07	4.23E+07	-5.13E+06	1.44E+10	2.60E+10	3.83E+09	6.73E+06	7.80E+09
DMU17	1.21E+08	1.98E+07	1.83E+08	3.52E+06	2.54E+10	4.00E+10	1.56E+08	2.89E+10	1.34E+10
DMU18	3.64E+08	1.45E+07	8.46E+07	5.71E+06	2.13E+10	3.60E+10	2.40E+07	4.08E+10	6.39E+09
DMU19	1.30E+07	6.06E+06	2.00E+08	2.47E+06	4.56E+10	2.74E+10	2.75E+07	1.78E+10	2.15E+10
DMU20	-1.26E+08	2.00E+06	2.52E+07	1.23E+06	1.24E+11	2.75E+10	6.26E+07	2.76E+03	4.71E+10

With the inputs and outputs in Table 3, each DMU could be on MPSS in five years horizon.

5. Conclusions and future works:

DEA is a leading approach for measuring efficiency, benchmarks and returns to scale. In this paper for the first time, a combination of DEA and ANN is used for estimating returns to scale and projection of inefficient bank branches on MPSS region. Conducting this new method on hundred branches of one of the Iranian commercial banks, we reached the forecasted benchmarks for inefficient branches, therefore management could programming the strategic plan. This may help branches to have goals and directions to achieve the goals.

This directions may be help authors for future researches: First, different forecasting methods can be used for estimating inputs and outputs instead of ANN. Second, various version of ANN can be utilize for estimating.

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