New Formulas for two and Three Dimensions Equally Spaced Array Antennas

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INTRODUCTION

For the new wireless communication technologies, a communication system employing several antennas has been recognized as an appropriate manner to enhance the directivity of the system. In recent times, the phased array antennas (PAA) have taken up an important position in the wireless communication systems as a tracking beam antenna that can primarily be used for a proper beam steering system. They have mainly been used for wideband and narrowband applications such as satellite and radar communication system, respectively. Particularly, in the phase array the amplitude weights remain constant and only phases are changed as the beam is steered [1].

Formula of radiation pattern of single element antenna and linear equally spaced phased array are derived in several books such as Balanis and Rao [2-3]. It has been shown that beam pattern of a linear phased array is a function of frequency, phase difference between the elements (φ) and antenna spacing. There are lots of novel works done to optimize equally and unequally spaced linear array of antennas [4-10].

Authors in [4] have worked on pattern synthesizing unequally spaced antenna array by using novel Particle Swarm Optimization. Then inheritance learning particle swarm optimization (ILPSO) was proposed which worked better than (PSO) to decrease the sidelobes. Authors in [5] have proposed a new unequally of slotted-waveguide antenna array. It was shown that with this unequally spaced, sidelobes go down as compare as equally spaced. A practical technique to synthesis of unequally spaced antennas was proposed by B. P Kumar, and G. R. Branner. They have shown that performance of this method is higher than the uniformly spaced arrays with the same number of antennas [6].

In this paper, the formulas are capable of calculating the beam pattern of a two and three dimensions (2 and 3-D) phased array in mathematical methods. Then the results are compared with the same array properties written program in Matlab, called as Array Calc V2.4 [11].

This paper is organized as follows. Overview of phased array antenna system is described in the next section. Then a section deals with linear equally spaced array of antenna for tracking a source has been discussed.

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A large part of this paper, Sections 3 and 4, discuss the derived formulas of two and three dimensions phased array antennas in mathematical model, respectively. To assess the validity of the proposed approach, the simulation results of these mathematical models with some analysis are presented in Section 5. Finally, conclusions are drawn in Section 6.

A. Background:

![Fig. 1: Magnetic field at a point P from a dipole antenna.](image)

The radiation field due to a dipole at distance \( r \) far from the dipole as it is indicated in Fig. 1 (Rao, 2000) is given by [3]:

\[
E = -\eta \beta I_0 \frac{d}{4\pi r} \sin \beta r \sin (\alpha t - \beta r) a_{\alpha} \\
H = -\eta \beta I_0 \frac{d}{4\pi r} \sin \beta r \sin (\alpha t - \beta r) a_{\alpha}.
\]

where \( \beta = \frac{\omega}{v_p} \) and \( \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\Omega = 377\Omega \) are propagation constant and free space wave impedance and \( v_p \) is the velocity of propagation of the wave that is equal to \( 3\times10^8 \text{ m/s} \).

An array of antennas of two elements consists of two dipoles is shown in Fig. 2 (Rao, 2000). They are located on the x-axis and parallel to the z-axis ((d/2, 0, 0) and (-d/2, 0, 0)). These two antennas are separated by the distance of \( d \) and observing a source at the point \( P \) (apart from antenna 1 and 2, \( r_1 \) and \( r_2 \) respectively). The current amplitudes of elements are assumed to be equal with a phase shift difference \( \alpha \). If the current of elements are indicated as \( I_1(t) \) and \( I_2(t) \) respectively, their formulas are given by [3]:

\[
I_1 = I_o \cos (\alpha t + \frac{\alpha \pi}{2}) \\
I_2 = I_o \cos (\alpha t - \frac{\alpha \pi}{2})
\]

Then the electric fields at point \( P \) from each element are achieved by applying equations (2) [3]:

\[
E_1 = -\eta_0 \beta I_0 d I_1 \sin \theta_1 \sin (\alpha t - \beta r_1 + \frac{\pi}{2}) a_{\alpha_1} \\
E_2 = -\eta_0 \beta I_0 d I_2 \sin \theta_2 \sin (\alpha t - \beta r_2 + \frac{\pi}{2}) a_{\alpha_2}
\]

![Fig. 2: An array of two elements with space of d.](image)
For \( r >> d \) (the point \( P \) is far from the array), in the amplitude factors the values of \( \rho_0 \) and \( \rho_j \) can be approximately equal to \( \rho \) then:
\[
\theta_j \approx \theta_0 \approx \theta \quad \text{and} \quad a_{\rho_0} \approx a_{\rho_j} \approx a_{\rho},
\]
and the values of \( r_1 \) and \( r_2 \) approximately can be equal to \( r \) (\( r_1 \approx r_2 \approx r \)), but in the phase factors, the value of \( \frac{d}{2} \cos \psi \) cannot be ignored, therefore [3]:
\[
r_1 \approx r - \frac{d}{2} \cos \psi \quad (6)
\]
\[
r_2 \approx r + \frac{d}{2} \cos \psi \quad (7)
\]
where \( \psi = \frac{\pi}{2} - \theta \) is shown in Fig. 2. Therefore, the total field would be [3]:
\[
E = E_1 + E_2 = \frac{-2\beta d I_0 \sin \theta}{4\pi} \left[ \sin \left( \alpha - \beta r + \frac{\beta d}{2} \cos \psi \right) + \sin \left( \alpha - \beta r - \frac{\beta d}{2} \cos \psi - \frac{\theta}{2} \right) \right] a_{\rho}
\]
Finally, \( E = -\frac{2\beta d I_0 \sin \theta}{4\pi} \cos \left( \frac{\beta d \cos \psi + \alpha}{2} \right) \sin \left( \alpha - \beta r \right) a_{\rho} \)

Equation (9) shows that the electric field of two antennas at point \( P \) is equal to the electric field of an individual element multiplied by [3]:
\[
2 \cos \left( \frac{\beta d \cos \psi + \alpha}{2} \right)
\]
Where equation (10) is called an array factor (AF) [1].
Thus the total electric field is equal to [1]:
Total electric field = Electric Field due to a single element \( \times \) Array Factor (11)
If the antennas were isotropic, the total radiation pattern of a 2-element array would be \( \cos \left( \frac{\beta d \cos \psi + \alpha}{2} \right) \) [3].

Equation (9) is shown that the array factor of array antennas is independent of the nature of each antenna, while carrying the same current and has the same distance. Therefore, any type of antenna can be replaced by dipole, whereas the formula of array factor and results can remain same [3].

**Fig. 3**: A linear of an equally spaced array, each filled circle indicates an individual antenna.

**B. Linear equally spaced array of antenna for tracking a source**

Fig. 3 (Rao, 2000) depicts \( n \)-number of antennas that forms a uniform spaced array. Each antenna carries the same current amplitude \( I_0 \) with progressive phase shift \( \alpha \) (\( I_0 \) cos\( \alpha \), \( I_0 \) cos\( \alpha + \theta \), \( I_0 \) cos\( \alpha + 2\theta \)…for antennas 1, 2, 3… respectively) [3].

For a point \( P \) far from the array(\( r > nd \)), far field can be acquired as follows (the elements are supposed to be identical as it was mentioned, thus the magnitude of radiators is identical to \( E_0 \)) [1]

At the point \( (r_0, \theta_0) \), the complex electric field due to the first element is \( 1e^{-j\theta_0} E_0 \), then the complex electric fields at that point due to elements 2,3,… are \( 1e^{j\alpha} e^{-j\theta_0-\alpha} E_0 \), \( 1e^{j2\alpha} e^{-j\theta_0-2\alpha} E_0 \),…. therefore the resultant field due to an array of antennas with \( n \)-element is [3]:
\[ \overline{E}(\psi) = l e^{-j\beta_0} + l e^{j\alpha} e^{-j\beta (t_0 - d \cos \psi)} + l e^{j2\alpha} e^{-j\beta (t_0 - 2d \cos \psi)} + \ldots + l e^{j(n-1)\alpha} e^{-j\beta (t_0 - (n-1)d \cos \psi)} \]
\[ \ldots + l e^{j(n-1)\alpha} e^{-j\beta (t_0 - (n-1)d \cos \psi)} \]
\[ = \left[ 1 + e^{j(\beta d \cos \alpha + \alpha)} + e^{j2(\beta d \cos \alpha + \alpha)} + \ldots + e^{j(n-1)(\beta d \cos \alpha + \alpha)} \right] e^{-j\beta_0} E_0 \]

where \( E_0 \) is far electric field at \( P \), due to an individual element \([1]\). Thus,

\[ A F = \frac{1 - e^{j(n(\beta d \cos \alpha + \alpha))}}{1 - e^{j(\beta d \cos \alpha + \alpha)}} e^{-j\beta_0} \]

The magnitude of \( AF \) is:

\[ A F = \left| \frac{1 - e^{j(n(\beta d \cos \alpha + \alpha))}}{1 - e^{j(\beta d \cos \alpha + \alpha)}} \right| \]

Then,

\[ AF = \frac{\sin n[(\beta d \cos \alpha + \alpha)/2]}{\sin[(\beta d \cos \alpha + \alpha)/2]} \]

Therefore, for a certain number of elements in an array, the beam pattern depends on the frequency, phase difference between the elements \( (\alpha) \) and antenna spacing.

**Fig. 4:** 2-D of an equally spaced array, each filled circle indicates an individual antenna.

II. 2-D equally spaced array of antennas for tracking a source:

Fig. 4 shows two-dimensional equally distanced \((d)\) array with the same current distributed and progressive phase shift \(\alpha\).

For the first line array, the complex electric field at the point \((r_0, \triangle)\) has been calculated in the previous section which is illustrated as \(\overline{E}_1(\psi)\) in equation (16):

\[ \overline{E}_1(\psi) = \frac{1 - e^{j(n(\beta d \cos \alpha + \alpha))}}{1 - e^{j(\beta d \cos \alpha + \alpha)}} e^{-j\beta_0} E_0 \]

This field is for a linear array, for calculating 2-D array; \(m\) indicates the number of rows and \(n\) shows the number of antennas in a row or is the number of column.

For the second row of array, the complex electric field at the point \((r_0 - d \sin \triangle, \triangle)\) is:

\[ l e^{-j\beta (t_0 - d \sin \psi)} e^{j\alpha} E_0 \]

then the complex electric fields at that point due to elements 2,3,...on the second row are

\[ l e^{j2\alpha} e^{-j\beta (t_0 - d \sin \psi - d \cos \psi)} E_0 , \ldots, l e^{j(2n-1)\alpha} e^{-j\beta (t_0 - d \sin \psi - (n-1)d \cos \psi)} E_0 \ldots. \]

Therefore, the field due to the \(n\)-element on the second row is:
Thus,  
\[
\vec{E}_2(\psi) = \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} e^{-j\beta t_0 / \sin \psi} e^{i\alpha} E_0
\]  
(18)

The complex electric field for the third row can be obtained as follows:

\[
\vec{E}_3(\psi) = \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} e^{-j\beta t_0 / \sin \psi} e^{i\alpha} E_0
\]  
(19)

For the mth row, the obtained complex electric field is:

\[
\vec{E}_m(\psi) = \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} e^{-j\beta t_0 / \sin \psi} e^{i\alpha} E_0
\]  
(20)

Therefore, the complex electric field due to n×m-element of a 2-D antenna array can be obtained as follows:

\[
\vec{E}(\psi) = \left\{ \leq e^{-j\theta} + e^{j\theta} e^{-j\beta t_0 / \sin \psi} e^{i\alpha} E_0 + \leq e^{-j\beta t_0 / \sin \psi} e^{i\alpha} E_0 + \ldots \right\}
\]  
(21)

Thus,

\[
AF_{mn} = \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} e^{-j\beta_0}
\]  
(21-a)

Or

\[
AF_{mn} = \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} e^{-j\beta_0}
\]  
(21-b)

The magnitude of (21-a) is:

\[
AF_{mn} = \left| \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} \frac{1 - e^{j\theta}}{1 - e^{j(\beta d / \cos \psi + \alpha)}} \right|
\]  
(22)

Let \( \theta_1 = \beta d / \sin \psi + \alpha \) and \( \theta_2 = \beta d / \cos \psi + \alpha \) So:

\[
AF_{mn} = \left| \frac{1 - e^{j\theta}}{1 - e^{j\theta}} \right| = \left| \frac{1 - e^{j\theta}}{1 - e^{j\theta}} \right|
\]
III. 3-D equally spaced array of antennas for tracking a source:

Fig. 5 shows a three-dimensional equally distanced (d) array with the same current distributed and progressive phase shift $\alpha$. For calculating 3-D array; $m$, $n$ and $o$ indicate number of rows, columns and planes respectively. Each element makes angles $\phi$, $\theta$, and $\gamma$ with $x$, $y$, and $z$ axes respectively, Fig. 6 (where $\cos^2 \phi \cos \theta \cos \gamma = 1$). It should be taken into account that $\psi \neq \frac{\pi}{2} - \theta$.

For the first plane array, the complex electric field at the point $(r_0, \phi, \theta)$ has been calculated in the previous section that is illustrated as $\bar{E}_1(\psi')$ in formula (25), Fig. 7:

$$\bar{E}_1(\psi') = \frac{1 - e^{j\beta(\theta_0 - d \cos \gamma)}}{1 - e^{j\beta(\theta_0 - d \cos \gamma)}} e^{-j\beta_0} E_0$$

In the second plane of array, Fig. 8, the complex electric field due to the first line can be achieved as follows:

The complex electric field at point $(r_0, \phi, \theta)$ is:

$$i e^{-j\beta(\theta_0 - d \cos \gamma)} e^{j\alpha} E_0$$

Then the complex electric fields at that point due to elements 2, 3, ... on the first row are

$$i e^{j2\alpha} e^{-j\beta(\theta_0 - d \cos \gamma - 2d \cos \gamma)} E_0, \quad i e^{j3\alpha} e^{-j\beta(\theta_0 - d \cos \gamma - 2d \cos \gamma)} E_0, \ldots, \quad i e^{j\alpha} e^{-j\beta(\theta_0 - d \cos \gamma - (n-1)d \cos \gamma)} E_0$$.

Therefore, the complex electric field of the first row in the second plane ($\bar{E}_{21}(\psi'))$ is:

$$\bar{E}_{21}(\psi') = i e^{j\alpha} e^{-j\beta(\theta_0 - d \cos \gamma)} E_0 + i e^{j2\alpha} e^{-j\beta(\theta_0 - d \cos \gamma - 2d \cos \gamma)} E_0 + \ldots + i e^{j\alpha} e^{-j\beta(\theta_0 - d \cos \gamma - (n-1)d \cos \gamma)} E_0$$

$$= \left[ 1 + e^{j(\beta_0 d \cos \gamma + \alpha)} + e^{j2(\beta_0 d \cos \gamma + \alpha)} + \ldots \right] e^{-j\beta(\theta_0 - d \cos \gamma)} e^{j\alpha} E_0$$

Fig. 5: 3-D of an equally spaced array, each filled circle indicates an individual antenna.
Fig. 6: Each element makes angles $\phi$, $\theta$ and $\psi$ with $x$, $z$ and $y$ axes respectively.

Fig. 7: First plane of 3-D array antennas, each filled circle indicates an individual antenna.

Fig. 8: Second plane of 3-D array antennas, each circle indicates an individual antenna.
Thus,

\[
E_{21}(\psi) = 1 - e^{jn(\beta d \cos \gamma + \alpha)} e^{-j(\delta_0 - d \cos \gamma)} e^{j\alpha} E_0
\]

(27)

The complex electric field for the second row can be obtained as below:

\[
E_{22}(\psi) = e^{j2\alpha} e^{-j(\delta_0 - d \cos \gamma - d \cos \theta)} E_0 + e^{j3\alpha} e^{-j(\delta_0 - d \cos \gamma - d \cos \theta - d \cos \psi)} E_0 + e^{j4\alpha} e^{-j(\delta_0 - d \cos \gamma - 2d \cos \psi)} E_0 + \ldots + e^{j(n+1)\alpha} e^{-j(\delta_0 - d \cos \gamma - (n-1)d \cos \psi)} E_0
\]

(28)

\[
= \left[ 1 + e^{jn(\beta d \cos \gamma + \alpha)} + e^{j2n(\beta d \cos \gamma + \alpha)} + \ldots + e^{j(n-1)\beta d \cos \gamma} + e^{j(n+1)\beta d \cos \gamma} \right] e^{-j(\delta_0 - d \cos \gamma)} e^{j2\alpha} E_0
\]

(29)

For the \(m\)th row, the obtained complex electric field is:

\[
E_{2m}(\psi) = 1 - e^{jn(\beta d \cos \gamma + \alpha)} e^{-j(\delta_0 - d \cos \gamma - (m-1)d \cos \theta)} e^{j\alpha} E_0
\]

(30)

Thus, the complex electric field due to the second plane will be obtained as follows:

\[
E_{2}(\psi) = 1 - e^{jn(\beta d \cos \gamma + \alpha)} - e^{j\alpha} E_0
\]

(31)

For the \(o\)th plane as it is shown in Fig. 9, the obtained complex electric field is:

\[
E_{o}(\psi) = 1 - e^{jn(\beta d \cos \gamma + \alpha)} - e^{j\alpha} E_0
\]

(32)

Therefore, the complex electric field due to \(n \times m \times o\)-element of a 3-D antenna array can be obtained as below:
\[ E(\psi) = \left[ e^{-\beta_\psi} + e^{i\alpha} e^{-\beta_i(\psi_e - \psi_{i-1})} + e^{2i\alpha} e^{-\beta_i(\psi_e - \psi_{i-1})} \right] + \]
\[ \ldots + e^{(i+2)\alpha} e^{-\beta_i(\psi_e - \psi_{i+1})} \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} - 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] E_0 \]
\[ = \left[ 1 + e^{i\beta_i(\psi_{i+1} - \psi_i)} + e^{2i\beta_i(\psi_{i+1} - \psi_i)} + \ldots \right] \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] E_0 \]
\[ A_F(\psi) = 1 - e^{(\alpha + \beta_i)\psi} \left[ 1 - e^{\beta_i(\psi_{i+1} - \psi_i)} \right] \]
\[ = 1 - e^{(\alpha + \beta_i)\psi} \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] e^{-\beta_i \psi} \]

Thus,
\[ A_F(\psi) = \left[ \frac{1}{1 - e^{(\alpha + \beta_i)\psi}} \right] \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] \]

The magnitude of (34) is:
\[ A_F(\psi) = \left| \frac{1}{1 - e^{(\alpha + \beta_i)\psi}} \right| \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] \]

Let \( \theta_1 = \beta \psi \cos \theta + \alpha \), \( \theta_2 = \beta \psi \cos \psi + \alpha \) and \( \theta_3 = \beta \psi \cos \psi + \alpha \)

So:
\[ A_F(\psi) = \left| \frac{1}{1 - e^{(\alpha + \beta_i)\psi}} \right| \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] \]

\[ = \sin \left[ \left( \beta \psi \cos \theta + \alpha \right) / 2 \right] \sin \left[ \left( \beta \psi \cos \psi + \alpha \right) / 2 \right] \]

\[ A_F(\psi) = \left| \frac{1}{1 - e^{(\alpha + \beta_i)\psi}} \right| \left[ 1 - e^{i\beta_i(\psi_{i+1} - \psi_i)} \right] \]

For the value of \( m \) and \( \alpha = 1 \), the same formula of equally spaced linear array (15) will be obtained.

**IV. Results:**

Results in this part are for 8 and 64-number of antennas in linear, 2 and 3-D arrays, for \( \alpha = 0^\circ \) and \( 180^\circ \) and \( 0^\circ \leq \psi \leq \pi \). From the results shown in Fig. 10, by increasing the dimension of an array of antennas with the same distance value \( d = 3 \lambda / 4 \), number and value of sidelobes go down while there is an increase in beamwidth.
Fig. 10: Normalized array factor \( F(\psi) \) versus \( \psi \) where \( \alpha=0^\circ \) and \( d=0.015 \text{ m} \) for (a) 8-element linear (b) 8-element 2-D (c) 8-element 3-D arrays.

Comparing Fig. 10 (b) with Fig. 11, beam width goes down by changing antenna spacing. It is shown that 2 and 3 dimensions with the same number of elements can give fewer numbers of sidelobes, less SLL and more beams than the linear array while the distance \( d \) should be wisely designed to get about the same directivity. (Multiple beams have benefit in communication such as mobile.)

Fig. 11: Normalized array factor \( F(\psi) \) versus \( \psi \) for 8-element 2-d array where \( \alpha=0^\circ \) and \( d=0.015 \text{ m} \).
Results in Fig. 12 indicate that with the increase the number of antenna, beamwith of the array will be narrower. Although the same beamwidth of a linear 8-element array can be obtained for a 2 and 3-D 8-element array by changing the value of $d$, but much has change differently for different change in number of elements.

Another factor that $AF$ can get change is $\alpha$, the difference in phase between the elements. In Fig. 13, $\alpha$ has the value of $180^\circ$. Comparing Fig. 10 to Fig. 13, for linear and 2 and 3-D arrays, the range of $\psi$ and its pick change with the change of $\alpha$ from $0^\circ$ to $180^\circ$.

**Fig. 12**: Normalized radiation pattern ($F(\psi)$) versus ($\psi$) where $\alpha=0^\circ$ and $d=\square$ for (a) 64-element linear array. (b) 64-element 2-D array. (c) 64-element 3-D array.
Fig. 13: Normalized array factor versus elevation angle ($\psi$) where $\alpha=180^0$ and $d=\frac{\lambda}{2}$ for (a) 8-element linear array (b) 8-element 2-D array. (c) 8-element 3-D array

To assess the validity of the proposed approach, the simulation results of these mathematical models with some analysis are compared to results using Array Calc V2.4 [11]. ArrayCalc is a tool that is based on a process of vector summation. This program is versatile and to compute the array patterns, its toolbox employs a
graphical method [11]. In order to compare the results, as it is seen in Fig. 14, array factors were multiplied by
the field of a single element positioned at the origin according to Balanis[2]. Results in Array Calc are achieved
for arrays of half-wave dipoles 0.25λ over a groundplane, antenna spacing 3λ/4 and α=0. The discrepancies of
SLL and Null to Null Beam Width are plotted in Fig. 15. According to the results achieved in Fig. 14 and 15,
mathematical formulas show valid results, although for more number of antennas and different spacing
discrepancies increases. Even though radiation patterns for a single elements are different in two modes and other
properties, results show that with the same numbers of antennas and spacing, higher dimension enhance SLL
and numbers of side lobes with wider null to null beam widths.
Fig. 14: Radiation pattern versus elevation angle ($\psi$) where $\alpha=0^\circ$ and $d=\lambda/4$ for (a) 8-element linear array in mathematical model and (b) its corresponding in Array Calc V2.4. (c) 8-element 2-D array in mathematical model and (d) its corresponding in Array Calc V2.4. (e) 8-element 3-D array in mathematical model and (f) its corresponding in Array Calc V2.4.
Fig. 15: (a) SLL curves of linear, 2-D and 3-D arrays (Solid and dash red lines indicate mathematical formulas used in study and Array Calc V2.4 model for a 8-element linear array where $\alpha=0^\circ$ and $d=\lambda/4$ respectively. Solid and dash blue lines indicate mathematical formulas used in study and Array Calc V2.4 model for 8-element 2-D array where $\alpha=0^\circ$ and $d=\lambda/4$ respectively. Solid and dash green lines indicate mathematical formulas used in study and Array Calc V2.4 model for 8-element 3-D array where $\alpha=0^\circ$ and $d=\lambda/4$ respectively.) (b) Calculated Null to Null Beam Width (Solid blue and dash red lines indicate mathematical formulas used in study and Array Calc V2.4 model for linear, 8-element 2-D and 3-D arrays where $\alpha=0^\circ$ and $d=\lambda/4$.)

Conclusion:
This paper has presented a formula for 2 and 3-D equally array antennas. From the results, it was shown that with the same number of antennas and space, directivity of linear array is better while sidelobes in 2 and 3-D is lower. With the change of distance, one can get the approximately as same directivity as linear array for a 2 and 3-D array with low sidelobes. With sagely design of 2 and 3-D array antennas, desire directivity with low sidelobes and less number of sidelobes could be achieved.

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