A Novel Nonlinear Programming Approach for Estimating CAPM Beta of an Asset Using Fuzzy Regression: A Case Study of BAHONAR Copper Company, the Company Listed on Tehran Stock Exchange

Hassan Dehghan Dehnavi and Soodeh Hajhoseiny

Department of Management, Yazd Branch, Islamic Azad University, Yazd, Iran

ABSTRACT

Evaluation of risky assets is one of the most important tasks of research in finance theory. Capital market theory is based on portfolio theory and aims to present a model that can price the risky assets. There are several capital assets pricing models the most well-known of which was proposed by Sharpe (1964) and Linter (1965). Capital assets pricing model claimed that there is only one agent namely market factor that affects return on assets. The main purpose of the current study was to investigate the relationship between market share, Tobin Q and return on assets. To this aim, fuzzy regression was performed and then fuzzy regression coefficients were obtained using LINGO software to solve corresponding linear programming problems. In this paper, market share and Tobin Q were taken as independent variables and return on assets was regarded as dependent variable. Non-random sampling method was used to collect data from firms listed in Tehran Stock Exchange for the years 2001 to 2010. Data analysis revealed that a fuzzy linear relationship exists between market share, Tobin Q and asset returns. After solving the model, values 0.665 and 0.04 were obtained for the coefficients, respectively.

INTRODUCTION

Since investors are uncertain about the future, they invest in different companies in order to reduce the investment risk; this is called portfolio. The investment risk can be reduced to the least degree through diversification of investment and proper combination of assets. The least degree of risk is dependent upon the correlation between combinatory assets.

Regardless of how much the portfolio is diversified, it is impossible to remove all the risks. From the perspective of an investor, any person is entitled to receive a rate of return that is worthy for the accepted risk. Capital asset pricing model (CAPM) helps to calculate investment risk and determine the expected rate of return.

A basic premise of the capital asset pricing model (CAPM) is that all agents invest in the portfolio with the highest expected excess return per unit of risk (Sharpe ratio) and leverage or de-leverage this portfolio to suit their risk preferences. However, many investors, such as individuals, pension funds, and mutual funds, are constrained in the leverage that they can take, and they therefore overweight risky securities instead of using leverage.[1]

However, the primary empirical challenge to the CAPM comes from several well-documented anomalies. A variety of managed portfolios constructed using various firm characteristics earn very different returns on average from those predicted by the CAPM.[2]

There are two fundamental factors in every investment that consist of the expected return and the risk that must be incurred to obtain the ROI; these two most important factors can be identified by means of CAPM. In fact, CAPM is a simple theory with a simple consequence in such a way that it even assures the risk averse investors that buying stocks with a greater risk than that of the market risk will lead to greater returns than the overall market return.

Corresponding Author: Hassan Dehghan Dehnavi, Department of Accounting, Yazd Branch, Islamic Azad University, Yazd, Iran. Safaiech, 8107 Shohadeh Gomnam Road. Zip code: 89195/155, Yazd, Iran. E-mail: s.hajhoseiny2013@yahoo.com
CAPM is based on the portfolio model proposed by Harry Markowitz in which risk and stock return is identified using the variance and average stock returns. CAPM was introduced by William Sharpe and John Lintner in 1960s; since then, the model has been widely used in different investment issues such as asset pricing, risk assessment, and portfolio performance measurement. According to this model, asset risk is measured by $\beta$ which is the systemic risk. When they want to make investment decisions, investors are only concerned with systematic risk since unsystematic risk is reduced through diversification. CAPM is shown in the following formula:[3]

$$R_t = \alpha_t + \beta_t R_{mt} + \epsilon_t$$

Where $R_t$ is the $i^{th}$ rate of return on asset for $t$ period, $R_{mt}$ is return on the market index for $t$ period; $\alpha_t$ is the fixed value for $i^{th}$ asset and $\epsilon_t$ is the error coefficient.

Under the null of a factor model, an asset’s expected excess return should be zero after controlling for that asset’s systematic factor exposure. Traditional regression tests of whether an alpha is equal to zero assume that the factor loadings are constant. However, overwhelming empirical evidence shows that factor loadings, especially for the standard capital asset pricing model (CAPM) and Fama and French models, vary substantially over time. Factor loadings exhibit variation even at the portfolio.[4]

Financing has always been one of the main reasons for the growth and development of businesses. Given the firms' need to provide liquidity, a variety of methods have been developed to finance each of which follow a certain pattern and structure; the identification of these patterns is essential.

Institutions and businesses, especially those active in the industry, need large capital to continue their existence and develop their activities; therefore, they become highly dependent upon the financial markets to fund for their capital requirements. The role of these markets is providing capital for institutions and companies.

The type of industry, organizational strategies, economic necessities, competition, major economic variables and financial indicators are among the most important factors that assist managers in making investment decisions about the allocation of resources to the most appropriate assets.

Companies need various resources to undertake profitable investment projects, settle debts, increase working capital and pay dividends to shareholders. These resources include cash from operating activities, asset sales, mortgages, issuing bonds and issuing new shares.

One of the important objectives of financial management is to maximize shareholders' wealth; thus, managers of economic sections should make crucial decisions about financing. Using appropriate forms of finance plays a key role in doing profitable projects that can increase shareholders' wealth.

Activists in financial knowledge have developed different theories, methods and models to achieve their purpose, that is to maximize shareholders wealth, have been developed. According to what mentioned above, conducting some studies on patterns of financing seems essential.

**Theoretical framework:**

Based on their financing policies, firms' financial resources can be divided into two parts: internal financial resources (retained earnings) and external financial resources (short-term and long-term debts and new issuance of stock).

Hierarchical theory of finance models was first introduced by Myers as follows:
1. Firms prefer internal financing sources.
2. A target dividend ratio is selected according to investment situations and sudden changes in dividends are avoided.
3. A fixed dividend policy with unexpected changes in profitability and investment positions is made, that means the internally generated cash flows are sometimes more and sometimes less than the capital costs.
4. In case external financing sources are required, the company will first issue the safest securities[5].

The ability of capital markets in financing the project could affect the firms' financial policy. If a company is always able to have access to foreign capital markets by the least expenses, there is no need for it to save cash. Also, if a company is faced with raising capital by increasing the expenses, it can increase its value by maintaining a more liquid balance sheet. Overall, every decision that affects the company's ability for financing its projects will also be affected over time by the distribution of financing demand and costs[6].

Ezeoha (2011) conducted a study about some Nigerian companies in which he investigated the set of factors determining capital structure that does not explain financing[7]. The results of Almeida’s (2011) study showed that companies have different types of investments in their economies with less developed financial markets.

Atherton (2012) observed in small businesses that financing patterns do not match with small business finance[8], while Alonso-Almeida (2012) showed that firms run by women started business with less initial capital and external financing than that of men[9]. Kim and Stock (2012) studied the banks and revealed that stabilizing the financial system led to the issuance of the excess of preferred stock by various banks[10].

Comparing two Jones' models based on fuzzy linear regression and least squares, Höglund (2013) found that as the length of the time series reduces, the performance of both models decreases. His findings also
revealed that no significant difference existed between the estimated discretionary accruals of the models. In addition, the Jones’ model which is based on fuzzy linear regression is an alternative for Jones’ least squares model. [11]

Using a semi-connected neural network model, Chang et al. (2012) showed that this model can be a very accurate prediction of stock price index for most of the data. Thus, it can be considered as a very good tool for predicting financial time series data.[12]

Goyal et al. (2014) showed that the least-squares method and fuzzy logic method can be used in modeling the daily evaporation of available climate data.[13]

Methodology:

This study used a descriptive correlational design and followed a practical application. Since the data used in this study were collected some years ago, it is considered as a type of causal comparative research.

Research hypotheses:

There is a significant relationship between Q-Tobin and return on assets.

There is a significant relationship between market share and return on assets.

Variables:

Variables in the current study consist of Q-Tobin and market share as the independent variables and return on assets as the dependent ones.

Q -Tobin ratio is among the indices that were first introduced by James Tobin, the Nobel winner in economics, to measure the value of companies. Nowadays, different forms of this index are widely used in financial management and venture capital in order to measure the value of listed companies on the Stock Exchange. This index represents ratio of market value of firms' assets to the replacement cost of the firms' assets.

The following formula is used in this study to calculate Q-Tobin.

\[ QTobin = \frac{MVE + ST + LT}{TA} \]

Where MVE is the market value of ordinary shares at end of year, ST is year-end book value of debt with a maturity of less than one year (short-term), LT is the book value of long-term debt at year-end, and TA is the book value of total assets at end of year.

Market share:

Market share is the part of the overall market which accounts for the institution and the institution's marketing programs that are designed, prepared and run to meet their marketing needs.

\[ MS = \text{Sales} \div \text{company's sales} \]

Return on Assets:

It is calculated by dividing the net profit after tax deduction in a given year by the average assets. This ratio reflects the efficiency of using assets and the amount of profit per Rial of the invested funds. The average of assets is calculated by total assets of the beginning and the end of the period divided by 2.

\[ ROA = \text{net profit} \div \text{average assets} \]

Data collection:

The data was collected from the study population that includes all firms listed in Tehran Stock Exchange during the years 2001 to 2010.

Non-random sampling was used to a company as a sample which had the following features.

1. The company must be listed on the Stock Exchange of Iran before 2006.
2. The end of their financial year must be March, 20 of each year.
3. The selected enterprise must not have any stoppage in their transaction or any change of the financial period.
4. The selected company must have the information needed for all 9-year period.

In this study, library data collection was used in which the required information for the literature review section was taken from Persian and Latin specialized online books, journals and papers. Some part of the data needed to test research hypotheses was derived from the financial statements of the companies under investigation. The other part of the information was taken from the Exchange databases; the desired tests were then performed by LINGO.

Linear regression mode with fuzzy coefficients:

Linear regression with fuzzy coefficients was first introduced by Tanaka et al. (1982). After their first article, numerous articles were published by same and others researchers about the above- mentioned theory
based on the development of theory and practical examples; among them, Heshmati and Kandle (1985) used it in forecasting.

It is assumed in the linear regression with fuzzy coefficients that the observations and the variables are precise and that only regression coefficients in the model are ambiguous. Suppose \( y \) is the dependent variable or the output, \( X_1, X_2, \ldots, X_p \) are the independent variables or inputs and \( n \) is the number of observations. Thereby, the general fuzzy linear regression model is as follows:

\[
\hat{Y} = \tilde{A}_0 + \tilde{A}_1 X_1 + \ldots + \tilde{A}_p X_p
\]

And it aims to estimate the model parameters, i.e. \( \tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_p \) to obtain the best estimation model for the data. To find the above parameters, the following triangular membership function is used:

\[
\tilde{A}(x) = \begin{cases} 
1 - \frac{a-x}{s} & a - s \leq x \leq a \\
1 - \frac{x-a}{s} & a < x \leq a + s 
\end{cases}
\]

Where \( a \) is the moderate amount, \( s \) and \( s^R \) are the lower bound and the upper bound of \( \tilde{A} \), respectively; that is \( \tilde{A} = (a, s, s^R) \). If \( s = s^L = s^R \), then \( \tilde{A} \) is a symmetric triangular number. Otherwise, it would be asymmetric. In the symmetric case, \( \tilde{A} \) is shown with \( r(a, s) \); in this case, the membership function of \( \tilde{A}(x) \) is as follows:

\[
\tilde{A}_j(x) = \begin{cases} 
1 - \frac{a-x}{s} & a - s \leq x \leq a \\
1 - \frac{x-a}{s} & a < x \leq a + s 
\end{cases}
\]

Estimating the model's parameters:

In the regression with fuzzy coefficients, the objective is that the coefficients \( \tilde{A}_i \) and \( i = 1, 2, \ldots, n \) is defined so that:

First, the fuzzy output \( \tilde{Y} \) has at least a degree of membership as large as \( h \) for all \( j = 1, 2, 3, 4, \ldots, \) and \( y_j \):

\[
\tilde{Y}(y) \geq h \quad j = 1, 2, 3, 4, \ldots, m
\]

Second, the model output has the least possible ambiguity or fuzziness. Therefore, to achieve the above objective, the total ranges of fuzzy outputs \( \tilde{Y}_j \) corresponding to all sets of data must be minimum to be called the objective function. This is shown in the following:

\[
Z = 2m(s_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij})
\]

In the above equation, \( X_{ij} \) is the \( j \)th observation of \( i \)th variable. Limitations of the model can be calculated in as follows.
It should be noted that two constraints must be generated for any observation; using linear programming techniques, the objective function $Z$ must be minimized with respect to $2m$ of the constraints generated by $m$ observation. For this purpose, different software can be used such as LINGO and GAMS.

Data Analysis:

Consider the following table that shows the data corresponding to Bahonar Copper Company. The first independent variable is market share, the second independent variable is $Q$-Tobin and the response variable is asset returns. The objective is to fit a linear regression model with fuzzy coefficients. According to what was discussed above, the objective function is written as follows:

$$Z= 10s_0 + s_1 \sum_{j=1}^{10} x_{j2} + s_2 \sum_{j=1}^{10} x_{j2} = 10s_0 + 0.032s_1 + 11.74s_2$$

<table>
<thead>
<tr>
<th>NO.</th>
<th>$y_i$</th>
<th>$x_{i1}$</th>
<th>$x_{i2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.87</td>
<td>20008154106</td>
<td>42.88</td>
</tr>
<tr>
<td>2</td>
<td>37.81</td>
<td>23134857433</td>
<td>37.81</td>
</tr>
<tr>
<td>3</td>
<td>36.92</td>
<td>90076676762</td>
<td>36.92</td>
</tr>
<tr>
<td>4</td>
<td>37.09</td>
<td>90066818702</td>
<td>37.10</td>
</tr>
<tr>
<td>5</td>
<td>63.46</td>
<td>612625831490</td>
<td>34.61</td>
</tr>
<tr>
<td>6</td>
<td>26.41</td>
<td>51345556061</td>
<td>26.42</td>
</tr>
<tr>
<td>7</td>
<td>26.62</td>
<td>58399975674</td>
<td>26.63</td>
</tr>
<tr>
<td>8</td>
<td>23.42</td>
<td>58495346858</td>
<td>23.43</td>
</tr>
<tr>
<td>9</td>
<td>27.30</td>
<td>65389074009</td>
<td>27.31</td>
</tr>
<tr>
<td>10</td>
<td>22.12</td>
<td>12789950542</td>
<td>22.12</td>
</tr>
</tbody>
</table>

The above table has 20 limitations.

Now, the above equation, which was written according to Bahonar Copper Company. The first independent variable is market share, the second independent variable is $Q$-Tobin and the response variable is asset returns. The objective is to fit a linear regression model with fuzzy coefficients. According to what was discussed above, the objective function is written as follows:

$$h = 0.1, 0.2, 0.3$$

$$0.9S_0 + 0.05S_1 + 1.386S_2 + 0.06S_1 - 1.54S_2 \geq 42.88$$

$$0.9S_0 + 0.05S_1 + 1.386S_2 - 0.06S_1 - 1.54S_2 \geq -42.88$$

$$0.9S_0 + 0.045S_1 + 1.035S_2 - 0.05S_1 + 0.05S_2 + 0.05S_2 - 1.15S_2 \geq -37.81$$

$$0.9S_0 + 0.048S_1 + 1.232S_2 - 0.06S_1 - 1.54S_2 \geq 42.88$$

$$0.9S_0 + 0.048S_1 + 1.232S_2 - 0.06S_1 - 1.54S_2 \geq -42.88$$

$$0.9S_0 + 0.04S_1 + 0.92S_2 - 0.05S_1 + 1.15S_2 \geq -37.81$$

$$0.8S_0 + 0.04S_1 + 0.92S_2 - 0.05S_1 + 1.15S_2 \geq -37.81$$

Considering the $h$ values and solving the above model, the values for coefficients were obtained indicating the best obtainable result for $h =0.5$. As mentioned before, the coefficient of each variable has two components ($a_i$, $s_i$):

The coefficients for $h=0.5$ are as follows:

$$(a_0, s_0) = (18.23, 4.298), (a_1, s_1) = (0.665, 0.309), (a_2, s_2) = (0.04, 0.0289)$$

Thus, there is a relationship between market share, Tobin $Q$ and return on assets. Therefore, the width and the facade are calculated using the following equations:

$$a_0 = 18.23 + 0.665x_1 + 0.04x_2$$

$$s_0 = 4.298 + 0.309x_1 + 0.0289x_2$$

Conclusion:

According to the proposed hypotheses and results, the following statements may be concluded:

Hypothesis 1 and 2: results showed that there is a relationship between Tobin $Q$ and return on assets as well as market share and return on assets. In the other words, Tobin $Q$ and market share can affect return on assets. As Tobin $Q$ and market share, which are actually variables of market performance, increase, return on assets also increases. For example, return on assets for the tenth year is calculated as follows:

$$a_n = 18.23 + 0.665 \times 0.02 + (0.04 \times 4) = 18.2833$$
\[
s_{r}=4.298+(0.309 \times 0.0289 \times 1)=4.6559
\]

Therefore, the level of return on assets in the tenth year in a fuzzy state can be represented as below.

\[
(18.2833-4.6559, 18.2833+4.6559) = (13.6274, 22.9392)
\]

As can be seen, return on assets is placed in the desired range.

REFERENCES