Advances in Environmental Biology, 9(21) Special 2015, Pages: 48-55

AENSI Journals

Advances in Environmental Biology

ISSN-1995-0756 EISSN-1998-1066

Journal home page: http://www.aensiweb.com/AEB/

Application of Optimal Control Theory to a Mathematical Model of Alcohol Abuse with Education Campaign and a Therapeutic Treatment

Bundit Unyong

Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Phuket, Thailand 83000

ARTICLE INFO

Article history:
Received 23 July 2015
Accepted 25 August 2015
Available online 5 September 2015

Keywords:
Alcohol epidemic, Basic reproductive number, Optimal control, Equilibrium.

ABSTRACT

Background: Optimal control theory is applied to a modified SITR model, we use two controls function representing education campaign and a therapeutic treatment that to reduce the number of the infectious individuals, and to increase the number of susceptible individuals and treatment individuals, respectively. The objective function is based on a combination of minimizing the number of infected individuals and maximizing the number of susceptible and recovery individuals. **Results:** The optimal controls are obtained by solving the optimality system. **Conclusion:** However, the result from the numerical solutions of the models as shown by using education campaign and a therapeutic treatment supporting all alcohol consumers to quit drinking will be the best strategies and the results be compared for supporting the analytic results.

To Cite This Article: Bundit Unyong., Application of Optimal Control Theory to a Mathematical Model of Alcohol Abuse with Education Campaign and a Therapeutic Treatment. ***Adv. Environ. Biol., 9(21),*** *48-55, 2015*

INTRODUCTION

The World Health Organization estimates that about 140 million people throughout the world suffer from alcohol dependence with related problems, such as the people being sick, losing a job among a host of other things [3,9]. Alcoholism sometime knows as alcohol dependence is a disease that includes alcohol craving and continued drinking despite its negative effect on an individual's health, relationships and social standing [7]. Alcoholism has a higher prevalence among men, although in recent decades, the proportion of female alcoholics has increased [11]. Alcohol damages almost every part of the body and contribute to a number of human diseases including but not limited to liver cirrhosis, pancreatitis, heart disease, sexual dysfunction and can eventually be fatal [2,11]. It had not much of work have been done in the mathematical model of alcoholism as growing health problem, although some studies offered some mathematical approaches to understand the growing burden of alcoholism [2,8,10]. G.P Samanta *et al* have developed a mathematical model of alcohol abuse to identify the parameters of interest for further study with a view for informing and assisting policy-makers in targeting prevention and treatment resources for maximum effectiveness. The paper is organized as follows. In section 2, we present a mathematical model including two controls strategy that represent education campaign and a therapeutic treatment. The analysis of optimization problem is presented in section 3. In section4, we give a numerical appropriate method and the simulation corresponding results. Finally, the conclusions are summarized in section 5.

2. Mathematical Model:

In this section we start by a mathematical model of alcohol abuse was proposed by [1]. The adult human population is divided into four compartments, namely ,occasional drinkers with frequency $S(t)$, heavy drinkers with frequency $I(t)$, drinkers in treatment with frequency $T(t)$, temporarily recovered class with frequency $R(t)$ at time t. The model can present by the following set of differential equations;

Corresponding Author: Bundit Unyong, Department of Mathematics, Faculty of Science and Technology, Phuket Rajabhat University, Phuket, Thailand 83000
E-mail: bundit_23@yahoo.com

$$\frac{ds}{dt} = B - \mu S - \beta\frac{SI}{N} + \gamma R\frac{S}{N}$$
$$\frac{dI}{dt} = \beta S\frac{I}{N} + \alpha T\frac{I}{N} - (\mu + \delta_1 + \phi)I \qquad (1)$$
$$\frac{dT}{dt} = \phi I - \mu T - \mu R - \gamma R\frac{S}{N}$$
$$\frac{dR}{dt} = \sigma T - \mu R - \gamma R\frac{S}{N}$$

With initial condition; $S(0) > 0,\ I(0) \geq 0, T(0) \geq 0, R(0) \geq 0,\ N = S + I + T + R$,

Where the model parameters are described as the following:

B = Recruitment rate of occasional drinkers,
β = The rate of transmission from occasional drinkers to heavy drinkers,
α = The rate of transmission from drinker in treatment to heavy drinkers,
γ = The rate of transmission from recovered class to occasional drinkers,
μ = Natural death rate of population,
δ_1 = The death rate of heavy drinkers,
δ_2 = The death rate of drinkers in treatment,
ϕ = The proportion of drinkers who enter a therapeutic treatment,
σ = Recovery rate of drinkers in a therapeutic treatment.

The model consist of the following assumptions: the population is isolated and closed. It means that the total population size remaining constant where, $N = S + I + T + R$, all members of the population mix homogeneously, so each individual has an equal chance of becoming a heavy drinker. The heavy drinking is passed to occasional drinkers by adequate contact with heavy drinkers not in treatment.

3. Basic Properties of the Model

3.1 Invariant region;

Theorem 1 The feasible region Ω defined by $\Omega = \{(S,I,T,R) \in R_+^4 : 0 < N < \frac{B}{\mu}\}$,with initial conditions $S(0) > 0,\ I(0) \geq 0, T(0) \geq 0, R(0) \geq 0,$ is positive invariant.(proof by [1])

3.2 Positivity of solutions:

Theorem 2 Given that the initial conditions of system (1) are $S(0) > 0,\ I(0) \geq 0, T(0) \geq 0, R(0) \geq 0.$ $S(t), I(t), T(t), R(t)$ be positive for all $t > \bar{t}$ where $\bar{t} = \inf\{t>0: S(0) > 0,\ I(0) \geq 0, T(0) \geq 0, R(0) \geq 0\}$.(proof by [1])

The Basic Reproductive Number R_0:

We obtained a basic reproductive number by using the next generation method (van den Driessche and Watmough, 2002)[4]. By rewriting the equations (1) in matrix form;

$$\frac{dX}{dt} = F(X) - V(X) \qquad (2)$$

Where $F(X)$ is the non-negative matrix of new infection terms and $V(X)$ is the non-singular matrix of remaining transfer terms.

And setting;

$$F = \left[\frac{\partial F_i(E_0)}{\partial X_i}\right] \text{and } V = \left[\frac{\partial V_i(E_0)}{\partial X_i}\right] \qquad (3)$$

for all $i, j = 1,2,3,4$, be the Jacobean matrix of $F(X)$ and $V(X)$ at E_0.The basic reproductive number (R_0) is the number of secondary case generate by a primary infectious case (Andeson and May, 1991; van den Driessche and Watmough, 2002) or basic reproductive number is a measure of the power of an infectious disease to spread in a susceptible population. It can be evaluated through the formula;

$$\rho\left(FV^{-1}\right). \qquad (4)$$

Where FV^{-1} is called the next generation matrix and $\rho\left(FV^{-1}\right)$ is the spectral radius (largest eigenvalues) of FV^{-1}. Then we get the reproduction number R_0 where ; Basic reproductive number R_0 is the transmission coefficient from occasional drinker to heavy drinker divided by the sum of the natural death rate of the population, the drinking related death rate of heavy drinkers who are not in treatment and the proportion of individuals who enter treatment;

$$R_0 = \frac{\beta_1}{\mu + \delta_1 + \phi} \quad (5)$$

The equilibrium points for (S^*, I^*, T^*, R^*) are found by setting the right hand side of each equations of the system (1) equal zero, then we obtain two equilibrium points as follows;

3.1 Alcohol Free Equilibrium Point (E_0) *:*

In the absence of the disease in the community, there are $I = 0$, we obtained $E_0(S, I, T, R)$ where

$$S = \frac{B}{\mu}, I = 0, T = 0, R = 0$$

3.2 Alcohol Endemic Equilibrium Point (E_1) *:*

In case the disease is presented in the community, $I > 0$, we obtained, $E_1(S^*, I^*, T^*, R^*)$ where;

$$S^* = \frac{N\{(\mu+\delta_1+\phi)(N(\mu+\delta_2+\sigma)+\alpha I^*)\text{-}\alpha\phi I^*\}}{\beta(N(\mu+\delta_2+\sigma)+\alpha I^*)}$$

$$T^* = \frac{N\phi I^*}{(\mu+\delta_1+\phi)+\alpha I^*}$$

$$R^* = \frac{\beta N\gamma\phi I^*}{\mu\beta((\mu+\delta_1+\phi)+N(\mu+\delta_2+\sigma)+I^*)+\gamma\{(\mu+\delta_1+\phi)N(\mu+\delta_2+\sigma)+(\mu+\sigma_1)\alpha I^*\}}$$

Next, substitute the values S^*, T^*, R^* into the first equation of (2) and simplify then we obtain; $aI^3 + bI^2 + cI + d = 0$ .where;

$$a = (\mu+\delta_1)\beta\alpha\{\gamma\phi - \alpha(\mu\beta + (\mu+\delta_1+\phi)\gamma)\},$$

$$b = (\mu+\delta_1)N(\mu+\delta_2+\sigma)\{(\mu\beta + N(\mu+\delta_2+\phi)\gamma)(\alpha\mu N\text{-}N(\mu+\delta_2+\sigma)\beta) - \gamma N\phi(\alpha\mu+\beta\sigma)\}$$
$$+\beta\alpha((\mu+\delta_1+\phi)N(\mu+\delta_2+\sigma) - B\alpha)\{\gamma\phi - (\mu\beta + (\mu+\delta_1+\phi)\gamma)\},$$

$$c = (\mu\beta+(\mu+\delta_1+\phi)\gamma)\{2BN(\mu+\delta_2+\sigma)\beta\alpha - (\mu+\delta_1+\phi)N^2(\mu+\delta_2+\sigma)^2\beta$$
$$+(\mu+\delta_1+\phi)N(\mu+\delta_2+\sigma)\mu N\alpha + (\mu+\delta)N(\mu+\delta_2+\sigma)\alpha\mu N\}$$
$$\text{-}N(\mu+\delta_2+\sigma)\gamma(\beta\alpha B\phi + (\mu+\delta_1+\phi)\alpha\mu N + (\mu+\delta_1+\phi)\beta\sigma\phi N),$$

$$d = N^2(\mu+\delta_2+\sigma)^2$$

Cleary d is positive, by using Descartes' rule of signs in equation $aI^3 + bI^2 + cI + d = 0$.

Therefore, if a is negative then there exists at least one positive value of I^* .

Finally, Routh-Hurwitz criteria is used for determining the stabilities of the model. . If $R_0 < 1$, then the alcohol free equilibrium point is local asymptotically stable: that is ,no have heavy drinkers with frequency, but if $R_0 > 1$, then the alcohol endemic equilibrium is local asymptotically stable. Optimal control is the standard method for solving dynamic optimization problems, when those problems are expressed in continuous time (Lenhart and workman,2006). In this paper, we have modified the alcohol abuse model proposed by [1]. By using this method as part of control measures for drinkers epidemics. Into the system of equations (1), we include two controls *u* and *v* that represent, respectively, the education campaign that reduces the numbers of heavy drinkers with frequency $I(t)$ and to increases the number of drinkers in a therapeutic treatment by frequency $T(t)$ with rate v , The mathematical system with controls is given by the nonlinear differential equations subject to non-negative initial conditions as the following;

$$\frac{ds}{dt} = B - \mu S\text{-}\beta(1-u)\frac{SI}{N} + \gamma R\frac{S}{N}$$
$$\frac{dI}{dt} = \beta(1-u)S\frac{I}{N} + \alpha T\frac{I}{N} - (\mu+\delta_1+\phi)(1-v)I \quad (6)$$
$$\frac{dT}{dt} = \phi(1-v)I\text{-}\mu T\text{-}\mu R\text{-}\gamma R\frac{S}{N}$$
$$\frac{dR}{dt} = \sigma T - \mu R - \gamma R\frac{S}{N}$$

With initial conditions; $S(0) > 0, I(0) \geq 0, T(0) \geq 0, R(0) \geq 0, N = S + I + T + R$.

Optimal control for the dynamics of drinking as an endemic; a mathematical model with dynamic behavior. In this section we use the optimal control theory to analyze the behavior of the system of equation (6). The objective one is to minimize the infected human and the other is to maximize the drinkers in treatment with frequency. Mathematically, for a fixed terminal times t_f, the problem is to maximize the objective functional;

$$J(u,v)=\int_0^{t_f}\left[S(t)+I(t)+\frac{B_1}{2}u^2(t)+\frac{B_2}{2}v^2(t)\right]dt \tag{7}$$

The parameter $B_1 \geq 0$ and $B_2 \geq 0$ denote weights that balance the size of the terms for a fixed terminal time t_f. Hence we are interested in finding an optimal control pair u^* and v^* ,such that:

$$J(u^*,v^*)=\min\{J(u,v):(u,v)\varepsilon U\} \tag{8}$$

Where, $U=\{(u,v):0\leq u,v\leq 1, t\in[0,t_f],\ \text{u } \textit{and } v \text{ are } \textit{Lebesgue measurable}\}$

Next, applying the Pontryagin's Maximum Principle, we derive necessary conditions for our optimal control and corresponding state variables, including the two control functions. Therefore we have four corresponding adjoint variables where λ_1 corresponds to S , λ_2 corresponds to I , λ_3 corresponds to T and λ_4 corresponds to R .

3.3 The Hamiltonian adjoint equations:

The Hamiltonian equation is formed by allowing each of the adjoint variables to correspond to each of the state variables accordingly and combining the result with the objective functional as below:

$$H=S(t)+I(t)+\frac{B_1}{2}u^2(t)+\frac{B_2}{2}v^2(t)+\sum_{i=1}^{4}\lambda_i f_i \tag{9}$$

Where f_i is the right hand side of the differential equation of the i^{th} state variables. The adjoint equations are formed by taking the derivative of the Hamiltonian with respect to each of the state variables as follow;By applying the Pontryagin's maximum principle [4] and the existence result of optimal control from [5], we obtain the following theorem:

Theorem 3:

There exists an optimal control $(u^*,v^*)\in U$,and corresponding solution $S^*,I^*,T^* and R^*$ that minimize $J(u,v)$ over U .And there exists adjoint functions $\lambda_1,\lambda_2,\lambda_3 and \lambda_4$ verifying;

$$\lambda_1'=-\frac{\partial H}{\partial S}=-1+\lambda_1(\mu-\frac{\alpha R(t)}{N}+\frac{\beta(1-u(t))I(t))}{N})-\lambda_2\frac{\beta(1-u(t))I(t)}{N}+\lambda_4\frac{\gamma R(t)}{N},$$

$$\lambda_2'=-\frac{\partial H}{\partial I}=\lambda_1\left(\frac{\beta(1-u(t))I(t)}{N}\right)-\lambda_2(\frac{\beta(1-u(t))I(t)}{N}+\frac{\alpha T(t)}{N}-(\mu+\delta_1+\phi)(1-v(t)))$$
$$-\lambda_3(\phi(1-v)-\frac{\alpha T(t)}{N})-1,$$

$$\lambda_3'=-\frac{\partial H}{\partial T}=-\lambda_2(\frac{\alpha I(t)}{N})+\lambda_3(\frac{\alpha I(t)}{N}+(\mu+\delta_2+\sigma))-\lambda_4\sigma,$$

$$\lambda_4'=-\frac{\partial H}{\partial R}=-\lambda_1\frac{\alpha S(t)}{N}+\lambda_4(\mu+\frac{\gamma S(t)}{N})$$

With the transversality conditions $\lambda_1(t_f)=\lambda_2(t_f)=\lambda_3(t_f)=\lambda_4(t_f)=0$, and the optimize control (u^*,v^*) is given by

$$u^*=\min\left\{1,\max\left\{0,\frac{((\lambda_2-\lambda_1)(\beta I^*S^*)}{NB_1}\right\}\right\}$$

$$v^*=\min\left\{1,\max\left\{0,\frac{(\lambda_3(\phi I^*)-\lambda_2(\mu+\delta_1+\phi))I^*}{B_2}\right\}\right\}.$$

Proof:

The existence of optimal control can be proved by using the results from [5]. The adjoint equations and transversality conditions can be obtained by using Pontryagin's Maximum Principle such that

$$\lambda_1'=-\frac{\partial H}{\partial S}=-1+\lambda_1(\mu-\frac{\alpha R(t)}{N}+\frac{\beta(1-u(t))I(t))}{N})-\lambda_2\frac{\beta(1-u(t))I(t)}{N}+\lambda_4\frac{\gamma R(t)}{N},$$

$$\lambda_2' = -\frac{\partial H}{\partial I} = \lambda_1\left(\frac{\beta(1-u(t))I(t)}{N}\right) - \lambda_2\left(\frac{\beta(1-u(t))I(t)}{N} + \frac{\alpha T(t)}{N} - (\mu+\delta_1+\phi)(1-v(t))\right)$$
$$-\lambda_3\left(\phi(1-v) - \frac{\alpha T(t)}{N}\right) - 1 \quad ,$$

$$\lambda_3' = -\frac{\partial H}{\partial T} = -\lambda_2\left(\frac{\alpha I(t)}{N}\right) + \lambda_3\left(\frac{\alpha I(t)}{N} + (\mu+\delta_2+\sigma)\right) - \lambda_4\sigma,$$
$$\lambda_4' = -\frac{\partial H}{\partial R} = -\lambda_1\frac{\alpha S(t)}{N} + \lambda_4\left(\mu + \frac{\gamma S(t)}{N}\right).$$

The optimal control pair (u^*, v^*) be obtained by finding the derivative of the Hamiltonian equation with respect to the control variables, equating to zero, and solving equation. Then we get;

$$\frac{\partial H}{\partial u} = B_1 u(t) + (\lambda_1 - \lambda_2)\frac{\beta IS}{N} \text{ ;Let } B_1 u(t) + (\lambda_1 - \lambda_2)\frac{\beta IS}{N} = 0$$

Then the optimal value for u is;

$$u^* = \frac{(\lambda_2 - \lambda_1)(\beta I^* S^*)}{NB_1}$$

And $\frac{\partial H}{\partial v} = \frac{\lambda_2(\mu+\delta_1+\phi)I - \lambda_3\phi I + B_2 v(t)}{B_2}$; Let $\frac{\lambda_2(\mu+\delta_1+\phi)I - \lambda_3\phi I + B_2 v(t)}{B_2} = 0$

Hence the optimal value for v is;

$$v^* = \frac{\lambda_3\phi I^* - \lambda_2(\mu+\delta_1+\phi))I^*}{B_2}$$

By the bounds in U of the control, the optimal control pair (u^*, v^*) is given by

$$u^* = \min\left\{1, \max\left\{0, \frac{(\lambda_2-\lambda_1)(\alpha_h I_\upsilon)S^*}{NB_1}\right\}\right\} \quad (10)$$
$$v^* = \min\left\{1, \max\left\{0, \frac{(\lambda_3(\mu_h+\gamma_h+\alpha_h) - \lambda_4\gamma_\upsilon(1-I^*_\upsilon - E^*_\upsilon))I^*}{B_2}\right\}\right\}.$$

For supporting analytic results we need to resolve the optimal control model numerically.

4. Numerical simulations:

In this section we present the results obtained by solving numerically from the following system;

$$\frac{\mathrm{ds}}{\mathrm{dt}} = \mathrm{B} - \mu\mathrm{S} - \beta(1-\mathrm{u}^*)\frac{\mathrm{SI}}{\mathrm{N}} + \gamma\mathrm{R}\frac{\mathrm{S}}{\mathrm{N}}$$
$$\frac{\mathrm{dI}}{\mathrm{dt}} = \beta(1-\mathrm{u}^*)\mathrm{S}\frac{\mathrm{I}}{\mathrm{N}} + \alpha\mathrm{T}\frac{\mathrm{I}}{\mathrm{N}} - (\mu+\delta_1+\phi)(1-v^*)\mathrm{I}$$
$$\frac{\mathrm{dT}}{\mathrm{dt}} = \phi(1-v^*)\mathrm{I} - \mu\mathrm{T} - \mu\mathrm{R} - \gamma\mathrm{R}\frac{\mathrm{S}}{\mathrm{N}}$$
$$\frac{\mathrm{dR}}{\mathrm{dt}} = \sigma\mathrm{T} - \mu\mathrm{R} - \gamma\mathrm{R}\frac{\mathrm{S}}{\mathrm{N}}$$
$$\lambda_1' = -\frac{\partial H}{\partial S} = -1 + \lambda_1\left(\mu - \frac{\alpha R(t)}{N} + \frac{\beta(1-u^*(t))I(t))}{N}\right) - \lambda_2\frac{\beta(1-u^*(t))I(t)}{N} + \lambda_4\frac{\gamma R(t)}{N} \;,$$
$$\lambda_2' = -\frac{\partial H}{\partial I} = \lambda_1\left(\frac{\beta(1-u^*(t))I(t)}{N}\right) - \lambda_2\left(\frac{\beta(1-u^*(t))I(t)}{N} + \frac{\alpha T(t)}{N} - (\mu+\delta_1+\phi)(1-v^*(t))\right)$$
$$-\lambda_3\left(\phi(1-v^*(t)) - \frac{\alpha T(t)}{N}\right) - 1 \quad ,$$

$$\lambda_3' = -\frac{\partial H}{\partial T} = -\lambda_2\left(\frac{\alpha I(t)}{N}\right) + \lambda_3\left(\frac{\alpha I(t)}{N} + (\mu+\delta_2+\sigma)\right) - \lambda_4\sigma,$$
$$\lambda_4' = -\frac{\partial H}{\partial R} = -\lambda_1\frac{\alpha S(t)}{N} + \lambda_4\left(\mu + \frac{\gamma S(t)}{N}\right)$$

With $S(0) = S_0, I(0) = I_0, T(0) = T_0 and R(0) = R_0$ and $\lambda_i(t_f) = 0, (i = 1,2,3,4)$

Since ,there were initial condition for the state variables and terminal conditions for the adjoints and the optimality system is two-point boundary value problem, with separated boundary conditions at $t = 0$ and t_f. Then we use the semi-implicit finite difference method to solve the optimality system (2).

Advances in Environmental Biology, 9(21) Special 2015, Pages: 48-55

We partition the interval $\left[t_0, t_f\right]$ at the point $t_i = t_0 + ih\left(i = 0,1,2,...,n\right)$, where h is the time step such that $t_n = t_f$. And we define the state and adjoint variable $S\left(t\right), I(t), T(t), R(t), \lambda_1, \lambda_2, \lambda_3, \lambda_4$ and the control u and v in terms of nodal points $S_i, I_i, T_i, R_i, \lambda_1^i, \lambda_2^i, \lambda_3^i, \lambda_4^i, u^i$ and v^i . Then we use a combination of forward and backward difference approximation as follows:

$$\frac{S_{i+1} - S_i}{h} = B - \beta\frac{\left(1-u_i\right)}{N}S_{i+1}I_i - \mu S_{i+1} - \alpha S_{i+1}R_i$$

$$\frac{I_{i+1} - I_i}{h} = \beta\frac{\left(1-u_i\right)}{N}S_{i+1}I_i + \alpha\frac{T_i I_{i+1}}{N} - \left(\mu + \delta_1 + \phi\right)(1-v^i)I_{i+1}$$

$$\frac{T_{\upsilon_{i+1}} - T_{\upsilon_i}}{h} = \phi I_{i+1}(1-v^i) - \alpha\frac{T_i I_{i+1}}{N} - (\mu + \delta_2 + \sigma)T_{i+1}$$

$$\frac{R_{i+1} - R_i}{h} = \sigma T_{i+1} - \mu R_{i+1} - \gamma\frac{R_{i+1}S_{i+1}}{N}$$

By using above technique, we approximate the time derivative of the adjoint variables by their first-order backward-difference as the following;

$$\frac{\lambda_1^{n-i} - \lambda_1^{n-i-1}}{h} = -1 + \lambda_1^{n-i-1}(\mu + \frac{\alpha R_{i+1}}{N} + \beta\frac{(1-u^i)}{N}I_{i+1}) - \lambda_2^{n-i}\beta\frac{(1-u^i)}{N}I_{i+1} + \frac{\lambda_4^{n-i}\gamma R_{i+1}}{N},$$

$$\frac{\lambda_2^{n-i} - \lambda_2^{n-i-1}}{h} = \lambda_1^{n-i-1}[\beta\frac{(1-u^i)}{N}S_{i+1}] - \lambda_2^{n-i-1}[\beta\frac{(1-u^i)}{N}S_{i+1} + \alpha\frac{T_{i+1}}{N} - (\mu + \delta_1 + \phi)(1-v^i)]$$

$$-\lambda_3^{n-i}[\phi(1-v^i) - \alpha\frac{T_{i+1}}{N}] - 1$$

$$\frac{\lambda_3^{n-i} - \lambda_3^{n-i-1}}{h} = \lambda_2^{n-i-1}\alpha\frac{T_{i+1}}{N} + \lambda_3^{n-i}\left(\alpha\frac{T_{i+1}}{N} + (\mu + \delta_2 + \sigma)\right) - \lambda_4^{n-i}\sigma$$

$$\frac{\lambda_4^{n-i} - \lambda_4^{n-i-1}}{h} = -\lambda_1^{n-i-1}\frac{\alpha S_{i+1}}{N} + \lambda_4^{n-i-1}(\mu + \frac{\gamma S_{i+1}}{N})$$

The algorithm for the approximation method to obtain the optimal control as follows;

Algorithm:

Step 1: $S\left(0\right) = S_0, I\left(0\right) = I_0, T(0) = T_0, R(0) = R_0, \lambda_i\left(t_f\right) = 0\left(i = 1,2,3,4\right)$ and $u(0) = v(0) = 0$

Step 2: For $i = 0,..., \mathrm{n}-1,$ do

$$\lambda_1^{n-i-1} = \frac{\lambda_1^{n-i} + h(1 + \lambda_2^{n-i}\beta\frac{(1-u^i)}{N}I_{i+1} - \lambda_4^{n-i}\frac{\gamma R_{i+1}}{N})}{h[\mu - \alpha R_{i+1} + \beta(1-u^i)I_{i+1}] + 1}$$

$$\lambda_2^{n-i-1} = \frac{\lambda_2^{n-i} + h\left(1 - \lambda_1^{n-i-1}\beta\frac{(1-u^i)}{N}S_{i+1} + \lambda_3^{n-i}(\phi(1-v^i) - \alpha\frac{T_{i+1}}{N})\right)}{1 - h\left(\beta\frac{(1-u^i)S_{i+1}}{N} + \alpha\frac{T_{i+1}}{N} - (\mu + \delta_1 + \phi)(1-v^i)\right)}$$

$$\lambda_3^{n-i-1} = \frac{\lambda_3^{n-i} + h(\lambda_2^{n-i-1}\alpha\frac{T_{i+1}}{N} + \lambda_4^{n-i}\sigma)}{1 + h(\alpha\frac{I_{i+1}}{N} + (\mu + \delta_2 + \sigma))}$$

$$\lambda_4^{n-i-1} = \frac{\lambda_4^{n-i} + h\lambda_1^{n-i-1}\alpha S_{i+1}}{1 + h(\mu + \frac{\gamma S_{i+1}}{N})}$$

$$M^{i+1} = \frac{(\lambda_2^{n-i-1} - \lambda_1^{n-i-1})(\beta I_{i+1})S_{i+1}}{NB_1}$$

$$T^{i+1} = \frac{(\lambda_3^{n-i-1}\phi I_{i+1}) - \lambda_2^{n-i-1}(\mu + \delta_1 + \phi))I_{i+1}}{B_2}$$

$$u^{i+1} = \min\left\{1, \max\{0, M^{i+1}\}\right\}$$

$$v^{i+1} = \min\left\{1, \max\{0, \mathrm{T}^{i+1}\}\right\}$$

End for

Step 3:

$S^*\left(t_i\right) = S_i, I^*\left(t_i\right) = I_i, T^*(t_i) = T_i, R(t_i) = R_i, u^*\left(t_i\right) = u^i$ and $v^*\left(t_i\right) = v^i$

End for

The simulations at endemic state were carried out using the following values taken from table1, and reproductive number $\mathrm{R}_0 = \frac{\beta_1}{\mu + \delta_1 + \phi}$ with initial condition; $S(0) = 0.13, I(0) = 0.5, T(0) = 0.5, R(0) = 0.6$ and results show below;

Advances in Environmental Biology, 9(21) Special 2015, Pages: 48-55

Table 1: Parameters values used in numerical simulation at endemic state.

Parameters	Description	Value
B	Recruitment rate of occasional drinkers	0.45
β	The rate of transmission from occasional drinkers to heavy drinkers	0.71
α	The rate of transmission from drinker in treatment to heavy drinkers	0.31
γ	The rate of transmission from recovered class to occasional drinkers,	0.01
μ	Natural death rate of population	0.026
σ	Recovery rate of drinks in treatment	0.1
ϕ	The proportion of drinkers who enter treatment	0.5
δ_1	The death rate of heavy drinkers	0.035
δ_2	The death rate of drinkers in treatment	0.03

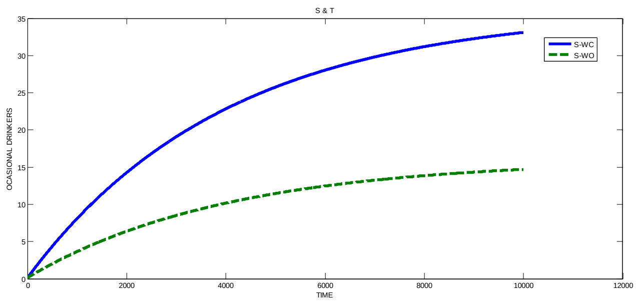

Fig. 1: Represent time series of susceptible individuals: occasional drinkers with frequency (S) with and without controls. It's show that the number of susceptible individuals (S) with controls increase after control.

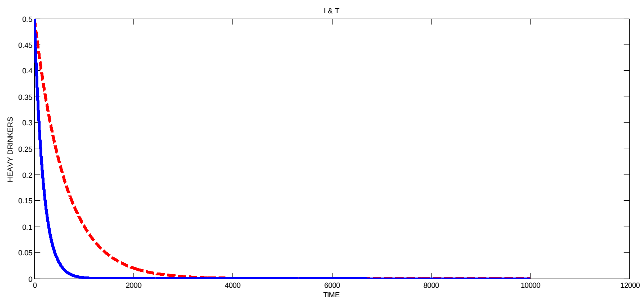

Fig. 2: Represent time series of heavy drinkers with frequency $I(t)$ with and without controls. It's show the number of heavy drinkers with frequency $I(t)$ with controls are decreased rapidly.

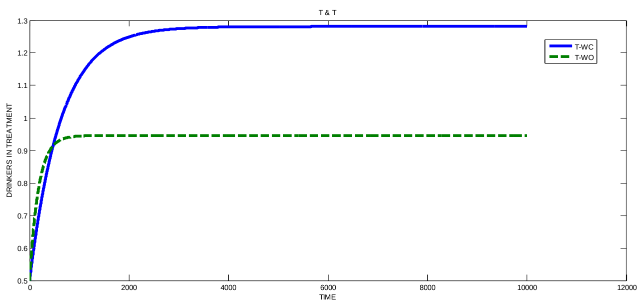

Fig. 3: Represent time series of drinkers in treatment with frequency $T(t)$ with and without controls. It's show that the number of drinkers in treatment with frequency $T(t)$ with controls are increased rapidly.

Advances in Environmental Biology, 9(21) Special 2015, Pages: 48-55

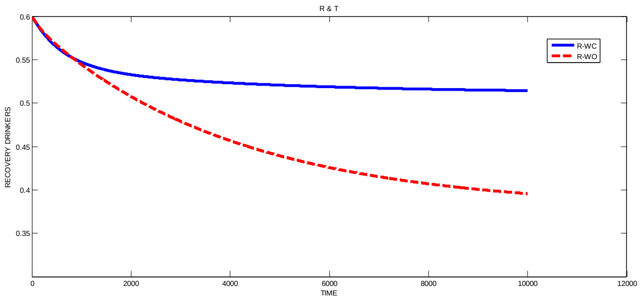

Fig. 4: Represent time series of temporarily recovered class with frequency $R(t)$ with therapeutic treatment and without controls. It's show the number of temporarily recovered class with frequency $R(t)$ with controls are increased more than the model with out control.

Conclusion:

In this paper, an modified SITR model for the transmission of optimal control theory to a mathematical model of alcohol abuse with education campaign and therapeutic treatment was proposed and analyzed. To reduce the contract between the susceptible human and the infected human and the other, minimizing the population of the infected human. The optimal control theory has been applied. By using the

Pontryagin's maximum principle, the explicit expression of the optimal controls was obtained. Simulation results indicate that after control by education campaign the numbers of occasional drinkers with frequency $S(t)$ are increased and the numbers of heavy drinkers with frequency $I(t)$ are decreased. But for the number of heavy drinkers with frequency are decreased after control.

ACKNOWLEDGEMENT

The authors are grateful to the Department of Mathematics, Faculty of Science and Technology and Phuket Rajabhat University, Thailand for providing the facilities to carry out the research.

REFERENCES

[1] Swarnali Sharma, G.P., 2013. samanta .Drinking as epidemic;Mathematical model with dynamic behavior. J. appl. Math, & information, 31(1-2): 1-25. Website: http://WWW.kcam.biz
[2] Blume, L., N. Nielson, 1998. Alcoholism and alcohol abuse among women: report of the council of scientific affairsJournal of Women's Health, 7(7): 861–870.
[3] Harlem-Brundtland, G., 2001. WHO European ministerial conference on young people and alcohol. World Health Organization, http://www.who.int/director-general/speeches/2001/english/20010219 youngpeople.en.html.
[4] Van den Driessche, P. and J. Watmough, 2002. Reproductive numbers and sub-threshold endemic equilibria for compartment models of disease transmission.Math. Biosci., 180: 29-48
[5] Pontryagin, L.S., V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko, 1962. The Mathematical Theory of Optimal Processes, Wiley, New York.
[6] Raj joshi, H., S. Lenhart, M.Y. Li, L. Wang, 2006. Optimal control methods applied to disease models-Contemporary Mathematics, 410.
[7] MedlinePlus, National Library of Medicine. National Institute of Health, 2009.http://www.nlm.nih.gov/medlineplus/ency/article/000944.htm.
[8] chowell, G., P.W. Fnimore, M.A. castillo-Chavez, 2003. sars out-breaks in Ontario,Hong kong and Singapore:the role of diagnosis and islation as a control mechanism,j. Theor.Biol, 224: 1-8.
[9] Riley, L.L., 2003. WHO to meet beverage company representatives to discuss health-related alcohol issues. World Health Organization, 37(3):129-132.
[10] Sanchez, F., X. Wang, 2007. Drinking as an epidemicła simple mathematical model with recovery and relapse. in:Guide to evidence-based relapse prevention, Elsevier, New York.
[11] Walter, H., K. Gutierrez, 2003. Gender specific differences in alcoholism: implications for treatment. Arch WomensMent Health, 6(4): 253–258.