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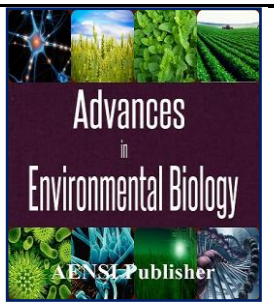

Effect of Environmental Hygiene Campaign on the Transmission of Cholera

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ABSTRACT

In this study, we proposed and analyzed a mathematical model to study the dynamics transmission of cholera with effects of environmental hygiene campaign as a control strategy. The model is analyzed using stability theory of differential equations and computer simulations. The results showed that there were two equilibrium points; cholera-free equilibrium point and cholera-present equilibrium point. The qualitative behavior results depend on the cholera reproductive number We obtained the cholera reproductive number by using the next generation method. Stabilities of the model are determined by Routh-Hurwitz criteria. If $R_0 < 1$, then the cholera- free equilibrium point is local asymptotically stable, but if $R_0 > 1$, then the cholera - present equilibrium point is local asymptotically stable. The graphical representations are provided to qualitatively support the analytical results. It concluded that with an increase in the effects of environmental hygiene campaign , the number of infected population will be decreased

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INTRODUCTION

Cholera is an acute bacterial infection of intestine caused by ingestion of food or water containing *Vibrio cholerae*. Symptoms include acute watery diarrhoea, vomiting which can result in severe dehydration or water loss [13]. If left untreated, an infected individual may become severely dehydration and die within two or three hours [9]. Cholera is transmitted through contaminated food or drinking-water as by person-to-person contact through the faecal - oral route. Sanitary conditions in the environmental play an important role since the *V. cholerae* bacterium survives and multiplies outside the human body and can spread rapidly where living conditions are crowded and water sources unprotected [13]. An estimated 3-5 million cholera cases and over 100,000 deaths occur each year around the world with more than 94% of cases in Africa. The infection is often mild or without symptoms, but can sometimes be severe. Approximately 5% infected individual will have severe disease characterized by profuse watery diarrhea, vomiting, and leg cramps [4].

Mathemaical models have become important tools in analyzing the spread and control of infectious diseases. Ochoche [9] proposed the mathematical model of cholera transmission using water treatment as a control strategy. The assumption of the model was the cholera is contacted only through the ingestion of contaminated water. Conditions of the existence of the disease free and endemic equilibrium point are derived and proved that the disease free equilibrium point is locally asymptotically stable under the given parameters. Numerical simulations are carried out to investigate the effect of water treatment on the dynamics of the infection. The numerical results showed that water treatment is an effective method of controlling cholera. Fakai *et al* [6] proposed the modification model of the previous cholera models. Model analysis was applied on the Jacobian matrix assuming zero *V. cholerae* environments. The basic reproductive number (R_0) and the critical number (R_c) were obtained.

These two values are used to predict occurrence of cholera outbreak in community. Posny and Wang [11] proposed a deterministic model for cholera dynamics in periodic environments. By incorporating seasonal variation into a general formulation for the incidence and the pathogen concentration. The basic reproductive number of the periodic model is obtained, based on which an analysis is conducted on the epidemic and endemic dynamics of cholera. The numerical results are used to support the analytic results. Agawal and Verma [1]

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proposed and analyzed a nonlinear delayed model with immigration for the spread of an infected disease cholera with carriers in the environment. The carrier population density is assumed to follow the logistic model and grows due to conductive human population density related factors. The model is analyzed by stability theory of differential equations and numerical results are presented by computer simulations. Both disease - free, carrier- free and endemic equilibrium are found and their stability investigated. The results showed that the spread of cholera increases due to growth of carriers in the environment and disease become more endemic due to immigration. Numerical simulations are carried out to investigate the effect of certain parameters on the spread of disease, to support the analytical results of the model. Cui, Wu and Zhou [5]. proposed a SVR-B model with imperfect vaccination. Model analysis is used to determine the local stability of both disease-free equilibrium and endemic equilibrium. We calculated the certain threshold condition known as the control reproductive number (R_V). If $R_V < 1$, we obtained sufficient conditions for the global asymptotic stability of the disease free equilibrium: the diseases with eliminated from community. If $R_V > 1$, the disease persists and the endemic equilibrium is global asymptotically stable. We performed sensitivity analysis of R_V on the parameters in order to determine the disease transmission and show that an imperfect vaccination is always beneficial in reducing disease spread within community. Isere, Osemwenkhae and Okuonghae. proposed two mathematical models that explained the dynamics of cholera in Nigeria. The first model investigated the bacteria population using a logistic definition for its growth in the expected habitat and their interaction with the susceptible population. The second model is an optimal control model that includes two time- dependent control functions with one minimizing the contact between the susceptible and the bacteria and the other, the population of the bacteria in the water. The results from the numerical solutions of the models presented showed that increasing the susceptible pool and the infected population above some threshold values were responsible for epidemic cholera. It also showed that the difference between the growth rate (r) and the loss rate (n) of the bacteria plays a huge role in the outbreak as well as the severity of the disease.

Khan *et al* [7] proposed a mathematical SIRB model which represents the dynamics of cholera. Stability of disease free equilibrium and endemic equilibrium are discussed. The disease free equilibrium is local asymptotically stable and globally when basic reproductive number less than one. Otherwise an unstable exist. If the basic reproductive number greater than one, local asymptotical the endemic equilibrium is stable. As well as global with some conditions. The numerical results of the model are shown to support the analytical results.

Panja and Mondal. [10] proposed an epidemic model associated with V. Cholera. In this model, there are two equilibrium points: disease free equilibrium point and endemic equilibrium point. If the basic reproduction number $R_0 < 1$, the disease free equilibrium is local asymptotically stable but the endemic equilibrium does not exist. When $R_0 > 1$, it is shown that the endemic equilibrium is global asymptotically stable under certain condition. Furthermore, the results showed how the socioeconomic status parameters play an important role on the spread of cholera disease.

Ochoche proposed a mathematical model for control of cholera transmission dynamics using treatment as a control strategy. The model is designed by dividing the system into classes leading to corresponding differential equations. The model is constructed on the assumption that cholera is contracted only though the ingestion of contaminated water. Conditions are derived for the existence of the disease free and endemic equilibria. We proposed that the disease free equilibrium is locally asymptotical stable under prescribed condition on the given parameters. This means that cholera can be eradicated under such conditions in finite times. Numerical simulations are carried out using parameters from published data to investigate the effect of transmission parameters on the dynamics of the infection. We simulated cases with no control, week and strong control. Our results showed that water treatment is an effective method of controlling cholera however cholera cases will continue to be present in the population if the contribution of the each infected person to the aquatic environment and the contact rate with contaminated water is high.

The objective of the study is to determine the effects of education program on the dynamics transmission of giving up smoking model. The structure of this paper is organized as follows. In section 2, we present a formulation model with the influence of education program. In section 3, we analyze the model by using the stability differential equations theory, to determine both disease free and endemic equilibrium point, derive the basic reproductive number and investigate the stability of the model. In section 4, we simulate the numerical results of the model numerically, which support our analytic results. Finally, we summary the conclusions of our study in section 5.

Model Formulation:

For this study, we formulated (SIRB) model (Susceptible-Infected-Recovered-Contaminatedwater) for the dynamics transmission of Cholera. Let and denoted the susceptible, the infected, and the recovered human population, respectively. Let denoted the concentration of the Vibrios in the environment (contaminated water). The cholera model is combined the system of human populations and the environmental component are two groups of population which consist of human population and concentration of the Vibrios environment . The

diagram of the transmission of the Cholera disease as shown in Fig. 1

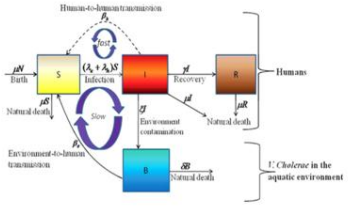

Fig. 1: Diagram of the transmission of the Cholera model.

The dynamical model can be represented by the following system of differential equations as follows.

$$\frac{dS}{dt} = \mu N - \frac{\beta_e S(1-c_w)B}{k+B} - \beta_h S(1-c_w)I - \mu S \qquad (1)$$

$$\frac{dI}{dt} = \frac{\beta_e S(1-c_w)B}{k+B} + \beta_h S(1-c_w)I - (\gamma+\tau+\mu)I \qquad (2)$$

$$\frac{dR}{dt} = (\gamma+\tau)I - \mu R \qquad (3)$$

$$\frac{dB}{dt} = x(1-c_s)I - \delta B \qquad (4)$$

$$S+I+R=N \qquad (5)$$

S is the number of susceptible individuals at time t
I is the number of infectious individuals at time t
R is the number of recovered individuals at time t
B is the number of *v. cholera* population at time t
μ is the natural death rate of human population
β_e is the environment-to-human transmission rate
β_h is the human-to-human transmission rate
k is the *v. cholera* infectious concentration
γ is the recovery rate from *v. cholera*
δ is the natural death rate of *v. cholera* population
x is the rate of contribution to *v. cholera* in the aquatic environment
c_w is the effectiveness of treated water
c_s is the effectiveness of save environment
N is the total number of human population .

From $N=S+I+R$

Consider

$$\begin{aligned}\frac{dS}{dt}+\frac{dI}{dt}+\frac{dR}{dt} &= \mu N-\beta_e S(1-c_w)\frac{B}{k+B}-\beta_h S(1-c_w)-\mu S+\beta_e S(1-c_w)\frac{B}{k+B}\\ &\quad +\beta_h S(1-c_w)I-\gamma I-\mu I-\tau I+\gamma I+\mu R\\ &= \mu N-\mu S-\mu I-\mu R\\ &= \mu N-\mu N\\ &= 0\end{aligned}$$

Thus, $\frac{dN}{dt}=0$, this means that the population is constant.

3. Model Analysis:

Equilibrium Points:

By using the standard method for analyzing our model, this system has two equilibrium points; clorela-free equilibrium point and cholera - present equilibrium point. We obtained these by setting the right hand side of equations, (1)-(4) to zero. Doing this, we obtain.

1. Cholera - Free - Equilibrium (CFE) denoted by $E_0(S,I,R,B)$

In the case of the absence of the disease, that is $I=0$. We obtained $S=N, R=0, B=0$. Thus, $E_0(S,I,R,B)=E_0(N,0,0,0)$.

2. Cholera - Present- Equilibrium (CPE) denoted by $E_1(S^*,I^*,R^*,B^*)$

In the case where the disease is present, that is $I\neq 0$. We obtained

$$S^*=\frac{\mu N(k\delta+x(1-c_s))}{x^2\beta_e(1-c_w)(1-c_s)+(1-c_w)\beta_h+\mu(k\delta+x(1-c_s))}$$

$$, I^*=\frac{-b+\sqrt{b^2-4ac}}{2a}, R^*=\frac{(\tau+\gamma)I^*}{\mu}, B^*=\frac{x(1-c_s)I^*}{\delta}.$$

where

$$a=(\gamma+\mu+\tau)(\beta_h x)(1-c_w)(1-c_s),$$
$$b=(\mu N\beta_h x)(1-c_w)(1-c_s)-(\gamma+\mu+\tau)(\beta_e x)(1-c_w)(1-c_s)$$
$$-(\gamma+\mu+\tau)(\beta_h k\delta)(1-c_w)-(\gamma+\mu+\tau)(\mu x)(1-c_s),$$
$$c=(\mu N\beta_h x)(1-c_w)(1-c_s)+(\mu N\beta_e k\delta)(1-c_w)-(\gamma+\mu+\tau)(\mu k\delta),$$

Cholera Reproductive Number:

The cholera reproductive number (R_0) (threshold condition in epidemiology) is the number of secondary infections induced by an infected individual introduced into the total susceptible population (Anderson and May,1991). By using the next generation method and used spectral radius (Van den Driessche and Watmough,2002). Doing this, we rewritten the system in matrix form.

$$\frac{dx}{dt}=F(x)-V(x) \quad , x=(S,I,R,B)^t$$

$$F(x)=\begin{bmatrix}0\\ \beta_e S(1-C_w)\frac{B}{k+B}+\beta_h S(1-C_w)I\\ 0\\ 0\end{bmatrix} \text{ and}$$

$$V(x)=\begin{bmatrix}-\lambda N+\beta_e S(1-C_w)+\beta_h S(1-C_w)I+\mu S\\ (\gamma+\mu+\tau)I\\ -(\tau+\gamma)I+\mu R\\ -x(1-C_s)I+\delta B\end{bmatrix}$$

Find the Jacobian matrix of $F(x)$ and $V(x)$ evaluated at $E_0=(N,0,0,0)$, we obtained,

$$F(E_0)=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \beta_h N(1-C_w) & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \text{ and } V(E_0)=\begin{bmatrix}\mu & \beta_h N(1-C_w) & 0 & 0\\ 0 & (\gamma+\mu+\tau) & 0 & 0\\ 0 & -(\tau+\gamma) & \mu & 0\\ 0 & -x(1-C_s) & 0 & \delta\end{bmatrix}$$

Find FV^{-1},we get

$$FV^{-1}=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & \frac{\beta_h N(1-C_w)}{(\gamma+\mu+\tau)} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

Thus, the spectral radius of FV^{-1} denoted by $\rho(FV^{-1})$

$$\rho(FV^{-1})=\frac{\beta_h N(1-C_w)}{(\gamma+\mu+\tau)}$$

We obtained the cholera reproductive number as shown,

$$\Re_0=\frac{\beta_h N(1-C_w)}{(\gamma+\mu+\tau)}$$

Stability Analysis:

In this section, we show the stability of the model at both disease free equilibrium and endemic equilibrium. First, we show that the system (1)-(4) is local asymptotically stable. The stability of this system as shown in the follow theorem.

Theorem 1:

The cholera- free equilibrium of the system (1)-(4) about the equilibrium E_0, is local asymptotically stable if $R_0 < 1$, and unstable if $R_0 > 1$.

Proof:

Since $R_0 < 1$, we have the Jacobian matrix of the system (1)-(4) at $E_0(N_T,0,0,0)$ is

$$J_0 = \begin{bmatrix} -\mu & -\beta_h N(1-c_w) & 0 & 0 \\ 0 & \beta_h N(1-c_w) - (\gamma+\mu+\tau) & 0 & 0 \\ 0 & (\tau+\gamma) & -\mu & 0 \\ 0 & x(1-c_s) & 0 & -\delta \end{bmatrix}$$

The eigenvalues of the Jacobian matrix J_0 are obtained by solving $\det(J_0 - \lambda I) = 0$. From this, we obtain the characteristic equation ,

$$(\lambda+\mu)^2(\lambda+\delta)\left(\lambda - \beta_h N(1-c_w) + (\gamma+\mu+\tau)\right) = 0$$

From the characteristic equation, we see that three eigenvalues are $\lambda_{1,2} = -\mu < 0, \lambda_3 = -\delta < 0$. The fourth eigenvalue will be negative if $(\beta_h N(1-c_w) < (\gamma+\mu+\tau)$.

Theorem 2:

The cholera - present equilibrium of the system (1)-(4) about the equilibrium E_1, is local asymptotically stable if $R_0 > 1$, and unstable if $R_0 < 1$.

Proof:

Since $R_0 > 1$, we have the Jacobian matrix of the system (1)-(4) at $E_1(S^*,I^*,R^*,B^*)$ is

$$J_1 = \begin{bmatrix} -\beta_e(1-c_w)\frac{B}{k+B} - \beta_h(1-c_w)I - \mu & -\beta_h S(1-c_w) & 0 & \beta_e S(1-c_w)\frac{k}{(k+B)^2} \\ \beta_e(1-c_w)\frac{B}{k+B} - \beta_h(1-c_w)I & \beta_h S(1-c_w) - (\gamma+\mu+\tau) & 0 & \beta_e S(1-c_w)\frac{k}{(k+B)^2} \\ 0 & (\tau+\gamma) & -\mu & 0 \\ 0 & x(1-c_s) & 0 & -\delta \end{bmatrix}$$

Where S^*, I^*, R^*, B^* are given by equation (4). The characteristic equation of Jacobian matrix at E_1, given by equations (1)-(4), becomes

$$(\lambda+\mu)(\lambda+\delta)^2\left(\lambda + \beta_e(1-c_w)\frac{B^*}{k+B^*} + \beta_h(1-c_w)I^* + \mu\right) = 0.$$

We obtain, the eigenvalues are

$$\lambda_1 = -\mu < 0, \lambda_{2,3} = -\delta < 0, \lambda_4 = -[\beta_e(1-c_w)\frac{B^*}{k+B^*} + \beta_h(1-c_w)I^* + \mu] < 0$$

Since all eigenvalues have negative real part which they satisfy the Routh-Hurwitz criteria (Marsden and McCracken,1976).

4. Numerical results:

The parameters used in the numerical simulation results are given in Table.1

Table 1: Parameter values in numerical simulations at disease free state.

Parameters	Descriptions	Values
μ	Birth (Natural death) rate of human population	0.000045 day^{-1}
β_e	Rate of ingesting *V. cholerae* from the contaminated water	0.99999 day^{-1}
β_h	Rate of ingesting *V. cholerae* thought human-to-human,	0.00011 day^{-1}
γ	Recovery rate of human population	0.5 day^{-1}
δ	Death rate induced of Cholera	0.3 day^{-1}
C_w	Effectiveness of treated water	0.099999
C_s	Effectiveness of save environment	0.01
τ	Effectiveness of treatment	0.5 day^{-1}
x	Rate of infected human contribution *V. cholerae* into the water	35 cells per ml day^{-1}
κ	Pathogen concentration that yields 50% chance of catching cholera	1,000,000 cells day^{-1}
N	Total number of population	6,000

Stability of disease free state:

Using the values of parameters as shown in Table. 1. We obtained the eigenvalues and the basic reproductive number as follows;

$\lambda_1 = -0.460052, \lambda_2 = -0.3, \lambda_{3,4} = -0.0000526, R_0 = 0.539972 < 1.$

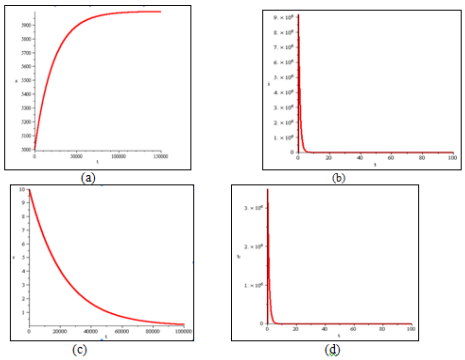

Fig. 2: Time series of (a) Susceptible population , (b) Infected population , (c) Recovered population and (d) Concentration of environment with the values of parameters;

$N = 6000, \mu = 0.000045, C_w = 0.99999, C_s = 0.01, \beta_e = 0.99999,$
$\beta_h = 0.00011, \gamma = 0.5, \delta = 0.3, \tau = 0.001, x = 35, \kappa = 100,000,$
$R_0 = 0.539972 < 1.$

We see that the solutions approach to the cholera- free equilibrium $E_0 = (6000, 0, 0, 0).$

Stability of endemic state:

We change the value of the effective of hot food consumption campaign to $\delta = 0.5$ and keep the other values of parameters to be those given in Table. 1. We obtain the eigenvalues and the cholera reproductive number as follows;

$\lambda_1 = -1.59099, \lambda_2 = -0.50018, \lambda_3 = -0.0000955203, \lambda_4 = -0.0000151388,$
$R_0 = 1.07969 > 1.$

Since all eigenvalues are to be negative and the cholera reproductive number is greater than one , the cholera-present equilibrium E_1 state, , will be local asymptotically stable as shown in Fig. 3.

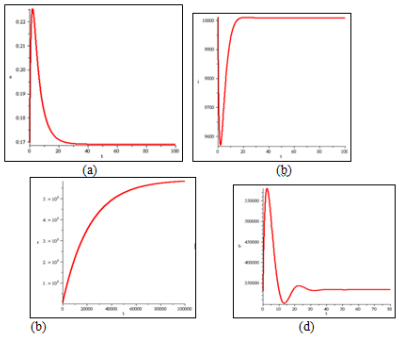

Fig. 3: Time series of (a) Susceptible population, (b) Infected population , (c) Recovered population and (d) Concentration environment with the values of parameters;

$N = 6000, \mu = 0.000045, C_w = 0.99999, C_s = 0.01, \beta_e = 0.99999,$
$\beta_h = 0.00011, \gamma = 0.5, \delta = 0.3, \tau = 0.001, x = 10, \kappa = 100,000$
, $R_0 = 1.07969 > 1.$

The state variables approach to cholera-present equilibrium

$E_1 = (0.169704, 100002.5, 1.11161 \times 10^5, 330082).$

Conclusion:

In this study, we proposed the mathematical model of Cholera with the effect of hot food consumption campaign and analyzed the analytical results by using standard modeling method. The cholera reproductive number is obtained through the use of spectral radius of the next generation matrix. The cholera reproductive number is $R_0 = \frac{\beta_h N(1 - C_w)}{(\gamma + \mu + \tau)}$. The cholera reproductive number is the threshold condition for determining the stability of the equilibrium points of the model which are shown in Fig. 2 and 3. Our simulation results shown

that R_0 will be decrease when the rate of infected human contribution *V. cholerae* into the water is decreased and the death rate of human induced cholera is decrease. We found the value of R_0 was 1.03908, when $x = 35$ and $\tau = 0.5$ and the value of R_0 was 0.73483 when $x = 10$ and $\tau = 0.001$, respectively. It seen that the infected human will decrease if the infected human be awareness about the infectious waste to contribute in the environment.

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